**PC-1690/M**

**L-4/2050**

COMMUTATIVE ALGEBRA-MM-710/AMC-419

(Semester–IV)

(Common for Math/AMC)

Time : Two Hours] [Maximum Marks : 70

**Note** : Attempt any *four* questions. All questions carry equal marks.

I. Let A be a finite ring :

(a) Prove that if $x \in$ A then some power of $x$ is idempotent.

(b) Verify that if $0 \neq e = e^2 \in$ A, then $1 - e$ is idempotent so can not be a unit.

(c) Deduce that N = R.

II. (a) Explain extension and contraction of ideals.

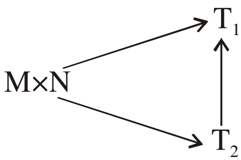
(b) Discuss tensor product of modulus.

III. Define tensor product if tensor product $M \otimes_R N$ are unique up to unique isomorphism then show that for two given tensor products :

$$r_1 : M \times N \to T_1 \text{ and } r_2 : M \times N \to T_2$$

there is a unique isomorphism

$i : T_1 \rightarrow T_2$ such that commutes, that is $T_2 = i$ or $T_1$.




IV. Define exactness properties of the tensor product. If P be any A - module then show that Hom (N, P) is also an A-module.

V. (a) Explain local properties of a ring.

(b) Discuss extended and contracted ideals in ring of fractions.

VI. (a) If Q is a primary ideal, then prove that the radical ideal $\sqrt{Q}$ is a prime ideal.

(b) If $a^n$ is in the prime ideal P, then prove that $a \in$ P.

VII. Let R be an h-local domain, and H its completion let P be a finitely generated, projective H module then P is isomorphic to a finite direct sum of principal ideals of H · P is a free H-module, if and only if rank mP is constant for all maximal ideals M of R.

VIII. (a) State and prove second uniqueness theorem.

(b) Discuss isolated prime ideals.

IX. (a) Let A = Z and $f : z \to Z$, defined as $f(z) = 2z$ then prove that $O \to Z \xrightarrow{f}$ is exact.

(b) Let $0 \neq e = e^2 \in R$, then show that $(1 - e)$ is idempotent.

(c) Explain tensor product of modules.

(d) Define Nil radical.

(e) Explain Zairiski topology.

(f) Prove that a prime ideal P of R is primary.

(g) Explain isolated prime ideals.

(h) Explain why is the localization at a prime ideal a local ring ?

(i) Define primary ideals.

(j) If $q \leq R$ is any primary ideal then prove that $r(q)$ is prime.