PC-3872/MR

G-9/2050

MECHANICS–II–602

Time : Two Hours] [Maximum Marks : 70

Note : Attempt any *four* questions. All questions carry equal marks.

I. (a) A particle is moving with uniform acceleration. In the eleventh and fifteenth second from the beginning, it travels 33 metre and 30 metres respectively. Find the initial velocity and the acceleration with which it moves. Find also the distance covered by it in 20 seconds.

(b) A point moves with uniform acceleration and v_1, v_2, v_3 denote the average velocities in successive intervals of time t_1, t_2, t_3; prove that $\frac{v_1 - v_2}{v_2 - v_3} = \frac{t_1 + t_2}{t_2 + t_3}$.

II. (a) A body falls freely from the top of the tower and during the last second of its flight it falls $\frac{16}{25}$th of the whole distance. What is the height of the tower?

(b) A particle projected vertically upwards takes time t_1 to reach a height h. If t_2 is the time to reach the ground again, prove that $h = \frac{1}{2} g t_1 t_2$ and the maximum height

attained is $\frac{1}{8} g (t_1 + t_2)^2$. Also, find the velocity of the particle at a height $\frac{h}{2}$.

III. (a) Two scale pans, each of mass m, are connected by a light string over a small pulley and in them are placed massed m_1 and m_2. Show that the reactions of the pans during motion are $\frac{2m_1 (m + m_2)}{m_1 + m_2 + 2m} g$ and $\frac{2m_2 (m + m_1)}{m_1 + m_2 + 2m} g$ respectively.

(b) A body sliding down a smooth inclined plane is observed to cover equal distances, each equal to l, in consecutive intervals of time t_1 and t_2. Show that the inclination of the plane is $\sin^{-1}\left[\frac{2l(t_1 - t_2)}{gt_1t_2(t_1 + t_2)}\right]$.

IV. A point moving in a straight line with simple harmonic motion has velocities u and v, and the corresponding accelerations are α, β. Show that the distance between two positions is $\frac{(v^2 - u^2)}{\alpha + \beta}$; time period of the motion is $2\pi\sqrt{\frac{u^2 - v^2}{\beta^2 - \alpha^2}}$ and its amplitude is $\frac{\sqrt{(u^2 - v^2)(\beta^2 u^2 - \alpha^2 v^2)}}{\beta^2 - \alpha^2}$.

V. (a) A uniform elastic string has length a_1 when the tension is T_1 and length a_2 when the tension is T_2. Show that its natural length is $\frac{a_2T_1 - a_1T_2}{T_1 - T_2}$ and the amount of work done in stretching it from its natural length to a length $a_1 + a_2$ is $\frac{(a_1T_1 - a_2T_2)^2}{2(T_1 - T_2)(a_1 - a_2)}$.

(b) A locomotive engine draws a load of m pounds up an inclined plane making at angle α with the horizontal, the coefficient of friction being μ. If starting from rest and moving with uniform acceleration, it acquires a velocity of v ft/sec in t seconds, show that the average horse power at which engine has worked is $\frac{mv}{1100}\left(\frac{v}{gt} + \mu \cos \alpha + \sin \alpha\right)$.

VI. (a) Two particle start simultaneously from A(–20, 0) and B(0, –15) and move along OX and OY respectively towards the origin with speed 15 m/sec and 20 m/sec respectively. Find the velocity of B related to A, one second after the motion begins.

(b) A train of mass M kg is ascending an incline of 1 in n, the resistance of motion being m kg weight per kg. of weight of train. The speed of the train is v metres/sec, when the power developed in the engine is H watts. Show that the acceleration is given by $\frac{nH - v^M g(1 + mn)}{Mnv}$ m/sec^2.

VII. (a) A bullet of mass m moving with a speed of v strikes a block of mass M which is free to move in the direction of motion of the bullet and is embedded in it. Show that a portion $\frac{M}{m+M}$ of the kinetic energy is lost.

(b) A uniform string of mass M and length $2a$ is placed symmetrically over a smooth peg and has particles of masses m and m' attached to its extremities. Show by principle of energy that when the string runs off the peg, its velocity is $\sqrt{\frac{M+2(m-m')}{M+m+m'}}\, ga$.

VIII. The masses of three spheres A, B and C are $7m$, $7m$, m ; their coefficient of restitution is unity, their centres are in a straight line and C lies between A and B. Initially A and B are at rest and C is given a velocity along the line of centres towards A. Show that it strikes A twice and B once and final velocities of A, B and C are proportional to 21,12,1.

IX. Attempt all the questions:

(a) Define simple harmonic motion and find its amplitude.

(b) Two masses m_1, m_2 ($m_1 > m_2$) are suspended by a light inextensible string passing over a smooth, fixed, light pulley. If the tension in the string is equal to the weight of mass M, prove that M is the harmonic mean between m_1 and m_2.

(c) Show that in rectilinear motion with constant acceleration, the distance described in successive seconds form an A.P.

(d) A particle is performing S.H.M. between two points A and B. If the period of oscillation is 2π, show that the velocity at any point P is mean proportional between AP and BP.

(e) If x_1 and x_2 be the distances described by the particle during the pth and $(p + q)$th seconds of its motion respectively, then prove that its acceleration is $\frac{x_2 - x_1}{q}$.

(f) If the time of one complete oscillation of a single pendulum is 20 seconds, find the length of the pendulum.

(g) Compute the angular momentum of Neptune about the sun given that it moves in a circular orbit of radius 5×10^{12} meter. Mass of Neptune is 10^{26} kg and its completes one revolution in 165 years.

(h) Define Work, Power and Energy.
