PC-3873/MR

G-9/2050

PARTIAL DIFFERENTIAL EQUATIONS–603

(Semester–VI)

Time : Two Hours] [Maximum Marks : 70

Note : Attempt any *four* questions. All questions carry equal marks.

I. (a) Solve the partial differential equation :

$$px(z - 2y^2) = (z - qy)(z - y^2 - 2x^3).$$

(b) Find the general solution of the linear equation :

$$(y + xz)p - (x + yz)q = x^2 - y^2.$$

II. (a) Solve the Cauchy's problem for $zp + q = 1$, when the initial data curve is

$$x_0 = \mu,\ y_0 = \mu,\ z_0 = \frac{\mu}{2},\ 0 \le \mu \le 1.$$

(b) Find the solution of the Pfaffian differential equation :

$$(y^2 + z^2)dx + xydy + zxdz = 0$$

III. (a) Find the equation of the surface which cuts orthogonally the family of cones $z^2 = c(x^2 + y^2)$, where $c \neq 0$ an arbitrary constant is. Obtain the particular surface which passes through the circle $z = 3$, $x^2 + y^2 = 9$.

(b) Solve the simultaneous equation :

$$\frac{dx}{z(x+y)} = \frac{dy}{z(x-y)} = \frac{dz}{x^2+y^2}$$

IV. Find the integral surface of $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$ which passes through the straight line $x = t$, $y = -t$, $z = 1$.

V. Find the complete integral of $p = (z + qy)^2$ by using Charpit method.

VI. Find the characteristics of the equation $z = p^2 - q^2$, and find the integral surface which passes through the parabola $4x + x^2 = 0$, $y = 0$.

VII. (a) Solve : $2\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} - 3\frac{\partial^2 z}{\partial^2 y} = \frac{5e^x}{e^y}$.

(b) Write down the canonical form of the one-dimensional wave equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$.

VIII. Find the solution of one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$, by the method of separation of variables, satisfying boundary conditions: $u(0, t) = 0 = u(l, t)$ and $u(x, 0) = (l - x)x$, $0 \le x \le l$.

IX. Attempt all the questions :

(a) Show that $p = (x + y)^2$, $q = x^2 + 2xy - y^2$ are compatible and solve them.

(b) Define Pfaffian differential equation in three variables.

(c) Write the Charpit's auxiliary equations for a differential equation $f(x, y, z, p, q) = 0$.

(d) Solve the partial differential equation $\frac{\partial^2 z}{\partial x \partial y} = 0$.

(e) Find the complete solution of partial differential equation $z = px + qy + \log(pq)$.

(f) State heat and wave equations for two variables.

(g) Write the Jacobi's auxiliary equations for solving the partial differential equations involving three independent variables.

(h) Classify the partial differential equation :

$$xy\, r - (x^2 - y^2)s - xy\, t + py - qx = 2(x^2 - y^2).$$

(i) Solve : $(D^2 + 2DD' + D'^2)z = e^{2x+3y}$ where $D = \partial/\partial x$ and $D' = \partial/\partial y$.

(j) Find the complete solution of $p + q = pq$.