PC-3875/MR

G-9/2050

TENSOR ANALYSIS-605

(Semester–VI)

Time : Two Hours] [Maximum Marks : 70

Note : Attempt any *four* questions. All questions carry equal marks.

I. (a) Find Cartesian coordinates of a point whose cylindrical coordinates are $\left(4, \frac{\pi}{3}, -2\right)$.

(b) Prove that ε_{ij} is a convariant tensor of rank 2.

II. (a) Prove that transformation of contravariant vectors forms a group.

(b) Prove that in an n-dimensional space, a symmetric covariant tensor of second order has atmost $n(n + 1)/2$ different components.

III. (a) If f is invariant, determine weather $\frac{\partial^2 f}{\partial x^p \partial x^q}$ is invariant.

(b) Show that the equation $x^1 = 4 \cos x^2$ is spherical co-ordinates represents a sphere.

IV. (a) Show that there is no distinction between contravariant and covariant vectors when the transformations are of type $\bar{x}^{i} = a^i_m x^m + d^i$, where d^i and a^i_m are constants such that $a^i_r \,.\, a^i_m = \delta^r_m$.

(b) If the relation $a_{ij}v^i v^j = 0$ holds for all vectors in v^i such that $v^i\lambda_i = 0$, where λ_i is a given covariant vector, show that $a_{ij} + a_{ji} = \lambda_i + \lambda_j v_i$.

V. (a) Find physical components of a vector with components A^i in spherical polar co-ordinates.

(b) Prove that $\frac{\partial}{\partial x^j}\left(\sqrt{g}\, g^{ij}\right) + \sqrt{g}\left\{ \begin{matrix} i \\ j \quad k \end{matrix} \right\} g^{jk} = 0$.

VI. Calculate the non vanishing Christoffel symbols of the second kind for $ds^2 = a^2(dx^1)^2 + a^2\sin^2 x^1 (dx^2)^2$.

VII. (a) If y^i and x^i are rectangular Cartesian and curvilinear co-ordinates, respectively, show that in E^3, $[ij, k] = \frac{\partial^2 y^p}{\partial x^i \partial x^j}\frac{\partial y^p}{\partial x^k}$.

(b) Show that the necessary and sufficient condition that the curl of a vector field vanishes is that the vector field be gradiant.

VIII. Find the coordinate surface defined by T : $x^1 = u^1 \cos u^2$, $x^2 = u^1 \sin u^2$, $x^3 = u^3$.

IX. (a) Define kronecker delta.

(b) If $a^i_j b^j_p = c^i_p$, then in matrix $\left(a^i_j\right)\left(b^j_p\right) = \left(c^i_p\right)$ and $\left|a^i_j\right|\left|b^j_p\right| = \left|c^i_p\right|$.

(c) Define equality of two tensors.

(d) Evaluate $\delta^j_i A^i$, range of indices 1 to 3.

(e) Show that if g^{ij} is a reciprocal tensor then $g_{ij}g^{kj} = \delta^k_i$.

(f) Express divergence theorem in tensor form.

(g) Prove that sum of two tensors of same kind is again a tensor of the same kind.

(h) Show that intrinsic differentiation of the product of two or more tensors satisfies the distribution laws.

(i) Show that number of independent components of Christoffel symbols are $n^2 \frac{(n+1)}{2}$.

(j) Prove that $[kj, i]$ are symmetric in j and k.