

Roll No.

Total Pages : 4

708/MH

C-2050

MATHEMATICAL METHODS-II

Option-iii

Semester-VI

Time allowed : 2 Hours] [Maximum Marks : 40

Note : Attempt any four questions. All questions carry equal marks.

1. (i) State and prove Parseval's identity for Fourier Transforms.

(ii) Using Parseval's identity for Fourier Transform to show that

$$\int_0^{\infty} \frac{\sin^2 as}{s^2} ds = \frac{a}{2}$$

2. (i) State and prove Convolution theorem of Fourier Transforms.

(ii) Use Convolution to find

$$F^{-1}\left(\frac{1}{-s^2 + 7is + 12}\right)$$

3. (i) Using Modulation theorem, find Fourier cosine transform of

$$f(t) = \begin{cases} 2 \cos 3t, & 0 \leq t \leq a \\ 0, & t > a \end{cases}$$

(ii) Find finite Fourier Sine Transform of

$$f(t) = \sinh at; a > 0, t \in (0, \infty)$$

4. (i) Find Fourier transform of $e^{-\frac{t^2}{2}}$

(ii) Show that $F_c(t \cdot f(t)) = \frac{d}{ds} F_s(s)$.

5. A string is stretched between two points (0, 0) and (1, 0). If it is displaced along the curve $y = k \sin \frac{x}{l}$, and released from rest in that position at time $t = 0$, find displacement $y(x, t)$ at any time $t > 0$ and at any point $x, 0 < x < l$.

6. Solve $\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}, 0 < x < l, t > 0$, where $y(x, 0) = 0$,

$$y_t(x, 0) = \sin \frac{x}{l}, y(0, t) = 0 \text{ and } y(l, t) = 0$$

7. (i) Solve $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = f(t)$

Where $f(t) = \begin{cases} 4t, & 0 < t < 1 \\ 8, & \text{otherwise} \end{cases}, y(0) = y'(0) = 0.$

(ii) Solve $t\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + ty = \sin t$

when $y(0) = 1, y'(0) = 2.$

8. Solve $\frac{dx}{dt} - 2y = t$ and $\frac{dy}{dt} - 4x + 2y = 0$

given $x(0) = 3, y(0) = 0$

9. (i) If $y(x, t)$ is defined for $x \in [a, b]$ where $t \geq 0$ and it is function of some exponential order as t tends to infinity and is sectionally continuous on each finite interval, then

$$L\left(\frac{y}{t}\right) = s\bar{y}(x, s) - y(x, 0)$$

- (ii) Find Fourier Transform of

$$f(t) = \begin{cases} e^{3it}, & a < t < b \\ 0, & t < a \text{ or } t > b \end{cases}$$

- (iii) Solve $f(t)$ satisfying $\int_0^\infty f(t) \cos t dt = e^{-s}$

(iv) Solve $\frac{y}{x} - \frac{y}{t} - 1 - e^{-t}, 0 < x < 1, t > 0,$

given that $y(x, 0) = x$

- (v) Define Finite Fourier Cosine Transform.

(vi) Solve $\frac{d^2y}{dt^2} + y = 0, y(0) = 0, y'(0) = 4$

- (vii) Find finite Fourier Sine transform of $f(t) = t^2.$

(viii) Solve $\frac{d^2y}{dt^2} - \frac{dy}{dt} - 6y = 5, y(0) = 0, y'(0) = 0$