BS/2110 5214/NH

ADVANCED CALCULUS
(PAPER I)
(SET II)

Time 3 Hrs
MM - 40

**Note: Attempt two questions from each of the Section A and B. Section C is compulsory.**

**Section - A** (6*2 = 12)

Q1.(a) If $f : R^2 \to R$ be defined by

$$f(x, y) = \begin{cases} 1, & \text{if } x \text{ is irrational}, \\ 0 & \text{if } x \text{ is rational} \end{cases}$$

Show that $f$ is not continuous at any point of $R^2$. (3)
(b) Show that for $f(x, y) = \frac{x^2y^2}{x^2y^2+(x-y)^2}$ the two repeated limits exist and are equal but the simultaneous limit does not exist. (3)

Q2.(a) State and prove Euler's Theorem on Homogeneous functions. (3)
(b) If $\sin^{-1}(\frac{x^2+y^2}{x+y})$. Prove that

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \tan z$$

and

$$x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = \tan^3 z$$

Q3(a) If $f(x, y) = (x + y, (x + y)^2)$. Evaluate $J_f(x, y)$. (3)
(b) Show that the functions $u = x - 2y + z$, $v = x^2 + 2xy - xz$, $w = 3x + 2y - z$ are not independent of one another. Also find the relation between u,v and w. (3)

Q4(a) Find the extreme values of $f(x, y) = x^3y^2(1 - x - y)$ (3)
(b) Find the shortest distance from the origin to the surface $lx+my+nz = p$ (3)

**Section - B** (6*2 = 12)

Q5(a) Evaluate $\int\int x^2y^2dxdy$ over the region $x^2 + y^2 \leq 1$ (3)
(b) Evaluate $\int\int\int(x+y+z)dxdydz$ over the tetrahedron bounded by $x = 0$, $y = 0$, $z = 0$, $x + y + z = 1$ (3)

Q6(a) Find the area enclosed by the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, $a > 0$
(b)Find the centre of gravity of the area bounded by the parabola $y^2 = x$ and the line $x + y = 2$ (3)

Q7(a) Evaluate $\int\int \frac{r}{a^2+r^2}drd\theta$ over one loop of the lemnisicate $r^2 = a^2\cos 2\theta$.
(b) Find the centre of mass of a solid cubical box bounded by the planes $x = -1$,

Contd ---2

$x = 1, y = -1, y = 1, z = -1, z = 1$ and density $\mu(x, y, z) = x^2$. (3)

Q8(a) $\int_0^a \int_0^a \frac{x}{x^2+y^2} dxdy$ Change the order of integration and hence evaluate the same. (3)

(b)Find the volume of the tetrahedron bounded by the coordinate planes and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, where a,b,c are positive. (3)

**Section - C** (2*8 = 16)

Q9. (a) Give an example of homogeneous function of degree 2.

(b) Find the percentage error in calculating the area of a rectangle when an error of 2 percent is made in measuring its sides.

(c) If a function $f(x, y)$ is differentiable at a point $(x_0, y_0) \in D_f \subset R^2$, then prove that it is continuous at that point.

(d) $z = x^3 - xy + y^3$, $x = r\cos\theta$, $y = r\sin\theta$. Find $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial \theta}$

(g) Evaluate $\int\int \sin\pi(x^2 + y^2)dxdy$ over the circle $x^2 + y^2 \leq 1$

(h)Find the area enclosed by the ellipse $\frac{x^2}{4} + \frac{y^2}{4} = 1$

(i) Find the moment of inertia of a square region of unit density about one of its sides, the side being 2a.

(j) ) If mass M is continuously distributed with density $\mu(x, y)$ over a region A of the xy plane. Write the formulae to calculate centeroid.

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