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5215/NH

Semester: III

Paper V: Analysis-I

Time: 3 Hours

Maximum Marks: 40

Instructions to Candidates:

Candidates are required to attempt five questions in all selecting two questions from each ***Section A*** *and* ***Section B*** *and compulsory question of* ***Section C****.*

Section-A

1. **(a)** Prove that a monotonically decreasing sequence converges if and only if it is bounded below. **3**

(b) Discuss the convergence and absolute convergence of the series

$$x-\frac{x^3}{3}+\frac{x^5}{5}-\ldots\ldots\ldots\ldots,x \text{ being real}$$ **3**

2. **(a)** If $\{a_n\}$ and $\{b_n\}$ be two convergent sequences such that $\lim_{n\to\infty} a_n = a,\ \lim_{n\to\infty} b_n = b$ then show that $\lim_{n\to\infty}\left(\frac{a_n}{b_n}\right)=\frac{a}{b},\quad b_n \neq 0, b \neq 0.$ **3**

(b) Test the convergence or divergence of the following series:

$$1+\frac{3}{7}x+\frac{3.6}{7.10}x^2+\frac{3.6.9}{7.10.13}x^3+\ldots\ldots\ldots\infty,\ x>0$$ **3**

3. **(a)** Apply Cauchy's general principle of convergence to show that $\{a_n\}$ where $a_n = 1+\frac{1}{2^2}+\frac{1}{3^2}+\ldots\ldots\ldots+\frac{1}{n^2}$ converges. **3**

(b) Show that the series $\frac{1}{2^3}-\frac{1+2}{3^3}+\frac{1+2+3}{4^3}-\frac{1+2+3+4}{5^3}+\ldots\ldots\infty$ is convergent. **3**

4. **(a)** Show that the series $1+\frac{\alpha}{1.\beta}x+\frac{\alpha(\alpha+1)^2}{1.2.\beta(\beta+1)}x^2+\frac{\alpha(\alpha+1)^2(\alpha+2)^2}{1.2.3\beta(\beta+1)(\beta+2)}x^3+\ldots\ldots\infty,$ $x>0$ converges when $x<1$ and diverges when $x>1$ and when $x=1$, the series converges if $\beta>2\alpha$ and diverges if $\beta\leq 2\alpha$. **3**

(b) Show that the sequence $\{x_n\}$ defined by $x_1=\sqrt{2},\ x_n=\sqrt{2x_{n-1}}$ converges to 2. **3**

Section-B

5. **(a)** If 'f' is R-integrable on $[a, b]$, then show that $|f|$ is also R-integrable on $[a, b]$ and $\left|\int_a^b f(x)\,dx\right| \leq \int_a^b |f|(x)\,dx$. **3**

(b) If f is monotonic on $[a, b]$, then show that f is of bounded variation on $[a, b]$. **3**

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6. Show that necessary and sufficient condition for a bounded function 'f' to be Riemann integrable on $[a, b]$ is that to every $\in > 0$, however small, there exists a partition P such that $U(P,f)-L(P,f)<\in$. . 6

7. (a) Prove that every continuous function is Riemann integrable. 3

(b) If f and g are each of bounded variation on $[a, b]$, then show that $f(x)+g(x)$ is also of bounded variation on $[a, b]$. 3

8. Let f be continuous on $[a, b]$. Then show that f is of bounded variation on $[a, b]$ if and only if f can be expressed as the difference of two increasing continuous functions. 6

Section-C (2 X 8 = 16)

9. (a) Prove that $\lim\limits_{n\to\infty}\left[\left(\frac{2}{1}\right)^1\left(\frac{3}{2}\right)^2\left(\frac{4}{3}\right)^3 \cdots\cdots\cdots\cdots \left(\frac{n+1}{n}\right)^n\right]^{\frac{1}{n}}=e$.

(b) If a sequence $\{a_n\}$ converges to l, then show that its subsequences $\{a_{2n+1}\}$ and $\{a_{2n}\}$ also converges to l.

(c) Show that the function 'f' defined by $f(x)=\begin{cases} x^2\cos\left(\frac{1}{x}\right) & ; x\neq 0 \\ 0 & ; x=0 \end{cases}$ is of bounded variation on [0, 1].

(d) Show that the series $\sum\limits_{n=1}^{\infty}\frac{1}{n^p}$ is convergent for $p>1$.

(e) Without evaluating, prove that $\frac{2}{17}\leq\int\limits_0^2\frac{1}{1+x^4}dx\leq 2$.

(f) Show that the function 'f' defined by $f(x)=\begin{cases} 0 \text{ ; when } x \text{ is rational} \\ 1 \text{ ; when } x \text{ is irrational} \end{cases}$ is not integrable on any interval.

(g) If the series $\sum a_n$ is absolutely convergent, then prove that $\sum a_n$ is convergent. Is the converse true?

(h) Let f be of bounded variation on $[a, b]$, let V be defined on $[a, b]$ as $V(x)=V_f(a,x)$ if $a<x\leq b$, $V(a)=0$. Then show that V is an increasing function on $[a, b]$.

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