CS/2110 [redacted] 1 5236/NH

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[redacted] .... Paper....I....

Title of the Paper ....Algebra-I....

Time allowed ....3 Hours....Maximum Marks ....40.... [redacted]

Note : The candidates are required to attempt two questions each from Section A & B Section C will be compulsory

Please assign marks to each que

SECTION-A

I (a) Show that the set of all positive Rational Numbers form an infinite abelian group under composition ■ defined as $a■b = \frac{ab}{2}$. (3)

I (b) Let G be a finite semi group with cancellation laws, then prove that G is group. (3)

II State and prove Lagrange's Theorem. (6)

III (a) Let G be a group such that G/Z(G) is cyclic, where Z(G) is centre of G. Then Prove that G is abelian. (3)

III(b) Let H and K be normal subgroups of a group G such that K is subset of H. Then Prove that $(G/K)/(H/K) \cong G/H$ (3)

IV (a) If G and G' are two groups and f is an isomorphism from G into G', then Prove that O(a) = O(f(a)) for all a ∈ G. (3)

IV (b) Let G and G' are two cyclic groups of same order, then prove that G and G' are Isomorphic to each other. (3)

SECTION-B

V (a) Prove that the set of rational numbers **Q** is a ring under the compositions $a \odot b = a + b - 1$ and $a \oplus b = a + b - ab \; \forall \; a, b \in \boldsymbol{Q}$ (3)

V (b) Prove that intersection of two subrings of a ring R is subring of R. Does the Result holds for union? (3)

VI(a) Let R and S be two rings. Then a Homomorphism $f: R \to S$ is one-one If and only if Ker f = {0}. (3)

VI (b) If I and J are two ideals of a commutative ring R with unity such that I + J = R. Then show that $IJ = I \cap J$. (3)

VII (a) For any two ideals I and J of a ring R, $I \cup J$ is an ideal of R iff either I is subset of J or J is subset of I. (3)

VII (b) Prove or disprove that there is an integral domain which has six elements. (3)

VIII (a) Prove that every Euclidean domain i.e. E.D. is principal ideal domain i.e. P.I.D. (3)

VIII (b) Let R be Euclidean Domain i.e. E.D. and a, b are any two non zero elements of R. If a is proper divisor of b, then d(a) < d(b). (3)

SECTION-C

IX (a) Let G be group of integers under addition and G' = {-1, 1} be group under Multiplication. Define a mapping $f: G \to G'$ as $f(x) = \begin{cases} 1 & if\ n\ is\ even \\ -1 & if\ n\ is\ odd \end{cases}$

Then prove that f is a Homomorphism

IX (b) Prove that a non empty subset H of a group G is a subgroup of G if and only if $ab^{-1} \in H \quad \forall\, a, b \in H.$

IX (c) Let G be a group and a be any element of G s.t. O(a) = n. Prove that $a^m = e$ if and only if n divides m.

IX (d) If H is a subgroup of a group G such that [G:H] = 2, then prove that H is normal Subgroup of G.

IX (e) Prove that every ideal of a ring R is subgroup of R, but converse is not true.

IX (f) Prove that centre of a Ring R is commutative subring of R.

IX (g) Prove that every field is Euclidean Domain i.e. E.D.

IX (h) Let R be a ring s.t. a ∈ R is idempotent i.e. $a^2 = a$. Then show that aRa is a subring of R. $2\times 8 = 16$