D-9/2110 5541/NJ

CSM-231 ADVANCE CALCULUS SEM 3^{rd})

Note : Do any 4 Questions

1. Prove that a Cauchy sequence is always Convergent
2. (a) If $b_n \leq a_n \leq c_n$ for all $n \in \mathbf{N}$ and $b_n \to l$, $c_n \to l$,Then prove that $a_n \to l$.
   (b) Prove that the Sequence $\{\frac{5n+1}{3n+5}\}$ is bounded .
3. Discuss the convergence or divergence of series Σa_n where $a_n = \frac{\sqrt{(n}+1)-\sqrt{(n-1)}}{n}$
4. If Σa_n is the positive terms convergent series , then show that $\Sigma {a_n}^2$ is convergent . Is the converse true? Justify your answer.
5. State and prove Abel'st's test on series. $(-1)^n$
6. Show that the series Σa_n converges where $a_n = (-1)^n \{\frac{(n+5)}{n(n+1)}\}$
7. If $f(x) = x^4-2x^3+2x^3-x$,Then prove by the Rolle's theorem that the equation $4x^3-6x^2+4x-1 = 0$ has atleast one real root in (0,1).
8. Show that the function $f(x) = \cos x$ is uniformly continous in the closed interval $[0,\frac{\pi}{2}]$
9. (a) Show that $\lim_{n\to\infty} (2^{-n}.n^2) = 0$
   (b) Show that the sequence $\{\frac{2n+7}{3n+8}.\}$ is convergent.
   (c) Give an example which shows that $\{x_n+y_n\}$ can be convergent without $\{x_n\}$ and $\{y_n\}$ being convergent
   (d) Define the Alternating Series and Alternating Series Test.

(e) Define Uniform Continuity of a function and prove that f(x) = x is Uniformly Continous

(f) Prove that a convergent sequence is always a Cauchy sequence.

(g) Find the value of k for which the function $f(x) = \begin{cases} x2 + 2x + k, & x \neq 0 \\ k, & x = 0 \end{cases}$

continuous at x=0