D-9/2110 5544/NJ

Maximum Marks: 30

Time: 02:00 hours

Statistical Inference I-234

(Sem-III)

Note: Students need to attempt any four questions in all. All questions will carry equal marks.

SECTION-A

1. Let X be distributed in the Poisson form with parameter θ. Show that only unbiased estimator of $e^{-(k+1)\theta}$, $k > 0$ is $T(x) = (-k)^x$ so that $T(x) > 0$ if x is even and $T(x) < 0$ if x is odd.
2. Describe the problem of point estimation and state some desirable properties of an estimator. Discuss how Neyman's factorization criterion helps in ensuring one of these desirable properties. Verify the same for the parameters of exponential distribution.
3. State the following terms
   a. Statistic
   b. Parametric space
   c. Shortest length confidence interval
   d. Sufficient Estimator
4. Examine whether the ratio $\frac{x}{n}$ where x denotes number of successes in n Bernoulli trials, is a consistent estimator of p.

SESCTION-B

5. Describe the method of maximum likelihood estimation. Derive the maximum likelihood estimators of μ and σ^2 based on a sample of size n from a $N(\mu, \sigma^2)$ distribution.
6. Describe the method of moments to estimate unknown population parameter using this method. Obtain an estimator of the parameter of Poisson distribution.
7. Define
   a. Two kinds of errors
   b. MP test
   c. Size of the test
   d. Critical region
8.
   a) Distinguish a composite hypothesis from a simple hypothesis giving examples

b) What is MLR property? Give an example of a family of distributions possessing this property.

SECTION-C

9.

a. Let $X_1, X_2, \ldots, X_n$ be iid Bernoulli variates $b(1, p)$, then show that $\frac{S_n}{n} \to p$ as $n \to \infty$ where $S_n = \sum_{i=1}^{n} X_i$.

b. Give an example to show consistent estimator of a population is not unique.

c. Let $X_1, X_2, \ldots, X_n$ be a random sample from exponential distribution with pdf $f(x, \theta) = \theta e^{-\theta x}$, $x > 0$. Using Neyam-Pearson lemma obtain the critical region for testing $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1$

$(2\frac{1}{2} + 2\frac{1}{2} + 2\frac{1}{2})$