D-10/2110 5552/NJ

CSM 352: Abstract Algebra
Semester 5

Time Allowed: 2 Hours **Maximum Marks: 45**

Note: - Attempt any ***four*** questions.Each question carries equal marks.

Q1.a) Prove that the set of all 2×2 matrices with entries from □ and determinant 1 is a group under matrix multiplication.
b) Check whether G = {2, 4, 6, 8} under multiplication modulo 10 is a Group ?

Q2.a) Write all symmetries of rectangle. Show that these symmetries form Klein 4-group
b)If H and K are subgroups of G and g belongs to G, then show that
$g(H \cap K) = gH \cap gK.$

Q3.a)Find the Kernel of the homomorphism $f: Z \longrightarrow Z_n$ given by $f(x) = \bar{x}$.
b) Show that $Z \times Z$is not cyclic group.

Q4.a) Give an example of a group G having subgroups H and K such that H is normal in K and K is normal in G but K is not normal in G.
b) Prove that every permutation of a finite set can be written as a cycle or as a product of disjoint cycles.

Q5.a) Let order of cyclic group is n. Show that if $d$divides$n$then number of elements of order d in a cyclic group of order n is $\emptyset(d)$.
b) If $f: G \longrightarrow G$ s.t $f(x) = x^n$is an automorphisms of G.Then show $a^{n-1} \in Z(G)\ \forall\ a \in G.$

Q6.If R is a commutative ring then show that the principal ideal

$$(a) = \{ar + na\ :\ r \in R, n \in Z\}$$

Q7.Let R be a PID which is not a field. Show that an ideal A is maximal if and only if A is generated by irreducible element.

Q8.Let R be a commutative ring with unity and A be an ideal of R. Then Show that

$$\frac{R[x]}{A[x]} \cong \frac{R}{A}[x]$$

Q9.Let $M = \left\{\begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix}\ : a \in R\right\}$ where R is any ring. Show that $\theta : M \longrightarrow$R s.t.
$\theta\begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} = a$ for all $\begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} \in M$ is an isomorphism.