D-12/2110 5572/NJ

MC 302/BMH 302 Group Theory

Semester 3

Time Allowed : 2 Hours **Maximum Marks : 70**

Note: - Attempt any ***four*** questions. Each question carries equal marks.

Q1.a) Prove that the set of all 2×2 matrices with entries from □ and determinant 1 is a group under matrix multiplication.
b) Prove that in a group, $(ab)^2 = a^2b^2$ if and only if $ab = ba$for all $a, b \in G$.

Q2.a) Prove that every subgroup of a cyclic group is cyclic. Moreover, also show that if order of cyclic group is n then order of subgroup divides n.
b)Let order of cyclic group is n. Show that if $d$divides$n$then number of elements of order d in a cyclic group of order n is $\emptyset(d)$.

Q3.a) List the elements of the subgroups $< 20 >$ and $< 10 >$ in Z_{30}. Let a be a group element of order 30. List the elements of the subgroups $< a^{10} >$ and $< a^{20} >$.
b) Let a and b belong to a group. If $|a|$ and $|b|$ are relatively prime, show that$< a > \cap < b > = \{e\}$.

Q4.a) Find a cyclic subgroup of A_8 that has order 4.
b)Prove that the set of even permutations in $S_n$forms a subgroup of S_n.

Q5.a) Verify Cayley's Theorem for group G = { 1, -1, i, - i}.
b) Show that U(8) is isomorphic to U(12).

Q6.a) Show that $Z \times Z$is not cyclic group.
b) If G is an abelian group and $f: G \longrightarrow G$ s.t. $f(x) = x^{-1}$ then show that $f$is an automorphisms of G. Is f an automorphisms of G when G is non abelian group ?

Q7.a) If H and K are subgroups of G and g belongs to G, then show that $g(H \cap K) = gH \cap gK$.
b) Is $Z_3 \oplus Z_9$ isomorphic to Z_{27}? Justify your answer.

Q8.a) Show that every group of order, $p^2$where p is a prime, is isomorphic to Z_{p^2} or$Z_p \oplus Z_p$.
b) Write all homomorphic images of Klein 4-group.

Q9.a) Prove that every permutation of a finite set can be written as a cycle or as a product of disjoint cycles.
b) How many abelian groups (up to isomorphism) are there of order 6?