5581/NJ

MC-501
Algebra (Group and Ring Theory)
Part-III (Sem- 5th)

Time Allowed: 2 Hours **Maximum Marks: 70**

Note: - Attempt any ***FOUR*** questions. Each question carries equal marks.

Q.1 Let H and K be two normalsubgroups of group G. Then Show that $\frac{G}{H \cap K}$ is solvable if and only if both $\frac{G}{H}$ and $\frac{G}{K}$ are solvable.

Q.2 Show that ring of integers has no composition series. Also verify Jordan Holder Theorem for Klein 4- group.

Q.3 Let R be a commutative ring with unity. Show that

i. If $a \in R$ is a unit then $a$is not nilpotent.
ii. If $a \in R$ is nilpotent then 1+ $a$is a unit.
iii. The sum of nilpotent and unit elements is a unit.

Q.4 Let R be a ring such that $x^3 = x, \ \forall x \in R$. Show that R is a commutative ring.

Q.5 Show that in a Boolean ring R, every prime ideal $P \neq R$ is maximal. Also show by an example that a finite commutative ring in which every maximal ideal need not be prime.

Q.6 Show that in ring $Z[\sqrt{-3}], 1 + \sqrt{-3}$ is irreducible but not a prime element.

Q.7 Show that every Euclidean Domain is a Unique Factorization Domain.

Q.8 Let R be a commutative ring with unity . Then Show that

$$\frac{R[x]}{< x >} \cong R$$

Also prove if R[x] is a PID then R is a field.

Q.9 Show that field of reals can be embedded into field of complex numbers. Also find the field of quotients of ring of Gaussian integers.