D-13/2110 5585/NJ

Discrete Mathematics and Graph Theory

(BHM-503 & MC-505)

SEM-V

Note: The question paper consists of NINE questions. The candidates are required to attempt any FOUR questions. All questions carry equal marks.

1. (a) State and Prove Pascal's formula. (6)
(b) State Pigeonhole principle. Use the Pigeonhole Principle to prove that an injectioncannot exist between a finite set A and a finite set B if the cardinality of A is greater than the cardinality of B.
(6)
(c) How many different five-digit numbers can be constructed out of the digits 1, 1,1, 3, 8?
(5.5)
2. (a) From the integers 1,2, ... ,200, if 101 integers are chosen. Show that, among the integers chosen, there are two such that one of them is divisible by the other. (6)
(b) State and prove Binomial theorem. (6)
(c) Show that if n + 1 integers are chosen from the set 1, 2, ..., 2n, then there arealways two which differ by 1.
(5.5)
3. (a) Use the binomial theorem to prove that $2^n = \sum_{k=0}^{n}(-1)^k\binom{n}{k}3^{n-k}$ (6)
(b) Determine the number of integral solutions of the equation $x_1 + x_2 + x_3 + x_4 = 20$that satisfy $1 \leq x_1 \leq 6;\ 0 \leq x_2 \leq 7;\ 4 \leq x_3 \leq 8;\ 2 \leq x_4 \leq 6.$
(6)
(c) A subway has six stops on its route from its base location. There are 10 peopleon the subway as it departs its base location. Each person exits the subway atone of its six stops, and at each stop at least one person exits. In how many ways can this happen? (5.5)
4. (a) Solve the recurrence relation $h_n = 5h_{n-1} - 6h_{n-2} - 4h_{n-3} + 8h_{n-4};\ (n \geq 4)$ withinitial values $h_0 = 0;\ h_1 = 1;\ h_2 = 1;\ h_3 = 2.$
(6)
(b) Let h_n denote the number of ways to color the squares of a 1-by-n board withthe colors red, white, blue, and green in such a way that the number of squarescolored red is even and the number of squares colored white is odd. Determine the exponential generating function for the sequence $h_0, h_1, h_2, h_3, \dots\dots\dots\dots$ and then find asimple formula for h_n.
(6)
(c) Solve the nonhomogeneous recurrence relation $h_n = 2h_{n-1} + n;\ (n \geq 1)$ withinitial value $h_0 = 1.$ (5.5)
5. (a) Let G be a connected graph of order 6 with degree sequence (2,2,2,2,2,2).
i. Determine all the nonisomorphic induced subgraphs of G,
ii. Determine all the nonisomorphic spanning subgraphs of G.

iii. Determine all the non isomorphic subgraphs of order 6 of G. (6)
(b) Does there exist a graph of order 5 whose degree sequence equals (4,4,3,2,2)? (6)
(c) Determine each of the 11 nonisomorphic graphs of order 4, and give a planarrepresentation of each. (5.5)
6. (a) Every connected graph has a spanning tree. (5.5)
(b) A multigraph is bipartite if and only if each of its cycles has evenlength. (6)
(c) State and prove Euler's formula. (6)
7. (a) A connected graph of order $n \geq 1$ is a tree if and only if it hasexactly n - 1 edges. (5.5)
(b) Prove that a planar graph is 5- colorable. (6)
(c)Draw two 3-regular graphs with
i. Eight vertices ii. Nine vertices (6)
8. (a) Let G be a connected planar graph. Then there is a vertex of Gwhose degree is at most 5. (6)
(b) Prove that the chromatic number of a disconnected graph is the largest of thechromatic numbers of its connected components. (6)
(c) Let G be a planar graph of order $n \geq 2$. Prove that G has at least two verticeswhose degrees are at most 5. (5.5)
9. (a) A chess master who has 11 weeks to prepare for a tournament decidesto play at least one game every day but, to avoid tiring himself, he decides not to playmore than 12 games during any calendar week. Show that there exists a succession of (consecutive) days during which the chess master will have played exactly 21 games.
(b) Find the recurrence relation for Fibonacci numbers.
(c) Every day a student walks from her home to school, which is located 10 blockseast and 14 blocks north from home. She always takes a shortest walk of 24blocks. How many different walks are possible?
(d) Determine the number of 12-combinations of the multiset s = {4. a, 3.b, 4. c, 5. d}.
(e) Draw a complete bipartite graph on two and four vertices.
(f) In a binary tree of height h, there are at most 2^{h-1} leaf nodes.
(g) Construct the graphs i) Eulerian but not Hamiltanion ii) neither Eulerian norHamiltanion. (7*2.5=17.5)