K-6/2110 7412-N

FIELD THEORY (MM602/AMC 309)

(Semester-3)

Time: 2 hours **M.M. 70**

The students are required to attempt four questions. All questions will carry equal marks.

Q.1 (a) State and Prove Eisenstein criterion. (9)

(b) Prove that a finite extension is finitely generated. Find the necessary and sufficient condition for a finitely generated extension to be finite extension . (8.5)

Q.2 (a) Find the splitting field of x^p-2, p prime over Q. Also find the degree of splitting field over Q. (9)

(b) Let F $\subset K \subset$ E be fields such that E is normal extension of K and K is normal extension of F. Is E normal extension of F? Justify your answer. (8.5)

Q3 (a) Let F be a field of order p^n, p prime. Prove that for each divisor m of n , there is unique subfield of F of order p^m. (9)

(b) Prove that the multiplicative group of non-zero elements of a finite field is cyclic group. (8.5)

Q.4(a) If E is a finite simple extension of F then prove that there are only a finite number of intermediate fields between F and E. (9)

(b) If F is a finite field of characteristic p, show that each element a of F has unique pth root in F.

(8.5)

Q5(a) Prove that the Galois group of $x^4 + 1$ over Q is Klein's four-group. (9)

(b) If f(x) is a polynomial over F with splitting field E and has r distinct roots in E, then prove that G(E/F) is a subgroup of symmetric group S_r. (8.5)

Q. 6 Let E be Galois extension of F and K be subfield of E containing F. Prove that K is a normal extension of F if and only for each $\sigma \in G(\frac{E}{F}), \sigma(K) = K$. Further prove that if K s a normal extension of F then G(E/K) is a normal subgroup of G(E/F). (17.5)

Q.7 Let F be a field of characteristic 0 and f(x) be a polynomial over F with splitting field E . Prove that if f(x) is solvable by radicals then G(E/F) is a solvable group. (17.5)

Q.8 Prove that the Galois group of $x^4 - 2$ over Q is the octic group and hence illustrate 1-1 correspondence between subgroups and subfields by using the Galois theory. (17.5)

Q.9(a) Find 10^{th} cyclotomic polynomial.

(b) Give an example each of cyclic and non-cyclic extension.

(c) Prove that any finite extension of finite field is simple.

(d) Let E be the splitting field of polynomial of degree n over field F then prove that [E:F]$\leq n!$.

(e) Express ${x_1}^2 + {x_2}^2 + {x_3}^2$ as rational function of elementary symmetric function.

(3.5×5=17.5)