K-6/2110

7413/N

Time: 03hrs. Differential Equations-II MM603/AMC-310

M.M:70

Section A

1. State and prove the existence and uniqueness theorem of solutions of first order differential equation $\frac{dw}{dz} = f(z, w)$ for complex systems.
2. State and prove Caratheodory theorem.
3. Let $f \in C$ $(n = 1)$ on the rectangular $0 \leq t \leq a, |x| \leq b$, where $a, b > 0$ and assume $f(t, x_1) \leq f(t, x_2)$ if $x_1 \leq x_2$ and $f(t, 0) \geq 0$ for $0 \leq t \leq a$. Prove that the successive approximations converge to a solution of $x' = f(t, x)$, $x(0) = 0$, on $0 \leq t \leq \alpha = min\left(a, \frac{b}{M}\right)$ where $M =$ max $|f|$ on the rectangle.
4. Let D be a domain of (t, x) space, I_μ the domain of $|\mu - \mu_0| < c$, $c > 0$ and D_μ the set of all (t, x, μ) satisfying $(t, x) \in D, \mu \in I_\mu$. Suppose f is a continuous function on D_μ bounded by constant M there. For $\mu = \mu_0$ let $x' = f(t, x, \mu)$, $x(\tau) = \xi$, have a unique solution ψ_0 on [a, b], where $\tau \in [a, b]$. Then prove that there exists a $\delta > 0$ such that, for any fixed μ, satisfying $|\mu - \mu_0| < \delta$, every solution I_μ of $x' = f(t, x, \mu)$, $x(\tau) = \xi$ exist over [a, b] and as $\mu \to \mu_0$, $\psi_\mu \to \psi_0$ uniformly over [a, b].

10x2=20

Section B

5. Show that the surfaces $x^2 + y^2 + z^2 = cx^{2/3}$ can form a family of equipotential surfaces, and find the general form of the corresponding potential function.
6. Find the distribution which gives rise to the potential
$$\psi = \begin{cases} a^2 - 3x^2; & r < a \\ \frac{a^5(y^2+z^2-2x^2)}{r^5} & ; r > a \end{cases} \quad \text{where } r^2 = x^2 + y^2 + z^2.$$
7. Find the potential function $\psi(x, y, z)$ in the region $0 \leq x \leq a, 0 \leq y \leq b, 0 \leq z \leq c$ satisfyig the conditions $\psi = 0$ on $x = 0, x = a, y = 0, y = b, z = 0$ and $\psi = f(x, y)$ on $z = c, 0 \leq x \leq a, 0 \leq y \leq b$.
8. A uniform insulated sphere of dielectric constant κ and radius $'a'$ carries on its surface a charge of density $\lambda P_n(\cos\theta)$. Prove that the interior of the sphere contributes an amount $\frac{8\pi^2\lambda^2 a^3 \kappa n}{(2n+1)(\kappa n+n+1)^2}$ to the electrostatic energy.

10x2=20

Section C

9. Write in brief.
   i). State maximum and minimum solutions of differential equation $x' = f(t, x)$ with $x(\tau) = \xi$.
   ii). Show that the continuity of a function f is not sufficient for the convergence of the successive approximations.
   iii). Define equipotential surface.
   iv). State hypothesis of the uniqueness of solution of differential equation $x' = f(t, x)$.
   v). State Copson's theorem.
   vi). Differentiate between the interior and exterior Dirichlet boundary value problem for Laplace equation.
   vii). State the Dirichlet's problem for a semi-infinite space.
   viii). State Green's theorem for Laplace equation.
   ix). State only existence theorem of solutions of first order differential equation for complex systems.
   x). Define inversion in a sphere.

10x3=30