K-6/2110 7419-N

Optimization Techniques-I
Paper: MM-609/AMC-316
(Semester III)

Time: 3 Hours **Maximum Marks: 70**

Note: Attempt any four questions. All Questions carry equal marks.

I (a) Solve Graphically the following LPP

$$\text{Min } z = 4x_1 + x_2$$
$$\text{Sub. to } 3x_1 + 4x_2 \geq 20,\ -x_1 - 5x_2 \leq -15$$
$$x_1,\ x_2 \geq 0$$

(b) Solve the L.P.P. using Two-Phase Simplex Method:

Minimize $z = x_1 + x_2 + x_3$
subject to constraints:
$x_1 - 3x_2 + 4x_3 = 5,\ x_1 - 2x_2 \leq 3,\ 2x_2 + x_3 \geq 4,$
$x_1, x_2 \geq 0$ and x_3 is unrestricted

II (a) Use Duality to solve the following LPP

$$\text{Maximize } z = 2x_1 + x_2$$
$$\text{Subject to } x_1 + 2x_2 \leq 10,\ x_1 + x_2 \leq 6,\ x_1 - x_2 \leq 2,\ x_1 - 2x_2 \leq 1$$
$$x_1,\ x_2 \geq 0$$

(b) Solve the following Integer Linear Programming problem using Branch And Bound Method:

Maximize $z = 2x_1 + 3x_2$
subject to constraints:
$x_1 + x_2 \leq 7, 0 \leq x_1 \leq 5, 0 \leq x_2 \leq 4,\ \ x_1, x_2$ are integers

III (a) A company has three operational departments (weaving, processing and packing) with capacity to produce three different types of clothes namely suitings, shirtings and woollens yielding a profit of Rs.2, Rs. 4 and Rs.3 per metre respectively. One metre of suiting requires 3 minutes in weaving, 2 minutes in processing and 1 minute in packing. Similarly one metre of shirting requires 4 minutes in weaving, 1 minute in processing and 3 minutes in packing. One metre of woollen requires 3 minutes in each department. In a week, total run time of each department is 60, 40 and 80 hours for weaving, processing and packing respectively. Formulate the linear programming problem to find the product mix to maximize the profit.

(b) Solve the following mixed integer programming problem.

Maximize $z = 4x_1 + 6x_2 + 2x_3$ subject to constraints

$4x_1 - 4x_2 \leq 5, -x_1 + 6x_2 \leq 5, -x_1 + x_2 + x_3 \leq 5$

$x_1, x_2, x_3 \geq 0$; x_1 and x_3 are integers

IV Given the LPP

Maximize $z = 3x_1 - 2x_2 + 4x_3$ subject to constraints:

$x_1 + 2x_2 + x_3 \leq 430, 3x_1 + 2x_3 \leq 460, x_1 + 4x_2 \leq 420$

$x_1, x_2, x_3 \geq 0$

BV	x_1	x_2	x_3	s_1	s_2	s_3	Sol
z	3	2	0	0	2	0	920
s_1	-1/2	2	0	1	-1/2	0	200
x_3	3/2	0	1	0	1/2	0	230
s_3	1	4	0	0	0	1	420

If we add $x_1 + x_3 \leq 200$ to the problem, find the new optimal solution of the modified problem.

V Given the following transportation problem:

	D	E	F	G	Supply
A	1	2	3	4	6
B	4	3	2	0	8
C	0	2	2	1	10
Demand	4	6	8	6	

Find the minimum cost programme. It is suspected that the solution is not unique. Find the alternative optimum solution, if any.

VI A small garments making unit has five tailors stitching five different types of garments. All the five tailors are capable of stitching all the five types of garments. The output per tailor and the profit (Rs.) for each type of garments are given below:

Tailors	Garments				
	1	2	3	4	5
A	7	9	4	8	6
B	4	9	5	7	8
C	8	5	7	9	8
D	6	5	8	10	10
E	7	8	10	9	9
Profit(Rs.)per garment	2	3	2	3	4

VI (a) Find the optimal strategies for both-persons and the value of game for zero-sum two-
I persons game graphically, whose payoff matrix is as follows:

$$\begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 6 \\ 4 & 1 \\ 2 & 2 \\ -5 & 0 \end{bmatrix}$$

(b) State and prove minimax and maximin principle of game theory.

VI Solve the following game by linear programming technique:
II

$$\begin{array}{c} \\ \\ player\ A \end{array} \begin{array}{c} player\ B \\ \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & -1 & 3 \\ 3 & 5 & -3 \\ 6 & 2 & -2 \end{bmatrix} \end{array} \begin{array}{l} \\ 1 \\ 2 \\ 3 \end{array}$$

IX (a) Define a convex set. Prove that $\{(x_1, x_2): 3x_1^2 + 2x_2^2 \leq 6\}$ is convex.

(b) Prove that dual of dual is primal.

(c) Find initial basic feasible solution by North West corner method to the following transportation problem:

		X	Y	Z	Available
Supplier	I	6	8	4	14
	II	4	9	8	12
	III	1	2	6	5
Requirement		6	10	15	31

(d) Explain the difference between transportation problem and assignment problem.

(e) What are characteristics and limitations of O.R. models?