[redacted] B-2110 Advanced Calculus 8461/NH

[redacted]. Sem III

Time: 3 hours M.M. 40

Candidates are required to attempt five questions in all selecting two from each of the section A and B and compulsory questions of section C.

Section-A

1. Prove that if $\lim_{(x,y)\to(a,b)} f(x,y)$, exists finitely then it is unique. (6)
2. State and prove Euler's theorem for homogenous functions. (6)
3. Expand $\sin(x+h)(y+k)$ by Taylor's Theorem. (6)
4. Find the maximum and minimum value of $x^2+y^2+z^2$ subject to the constraints $\frac{x^2}{4}+\frac{y^2}{5}+\frac{z^2}{25}=1, z=x+y$. (6)

Section-B

5. Evaluate the appropriate determinant to show that the Jacobian of the transformation from cartesian $\rho\emptyset\theta$ –space to cartesian xyz-space is $\rho^2 \sin\emptyset$. (6)
6. Compute $\iint_E \sin\left[\frac{x-y}{x+y}\right] dxdy$, where E is the region bounded by $x=0,\ y=0$ and $x+y=1$ in the first quadrant. (6)
7. Find the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. (6)
8. A cylinder rod of constant density lies along with the line segment $r(t)=tj+(2-2t)k$, $0\le t\le 1$, in the yz –plane. Find the moment of inertia and radius of gyration about the three coordinate axes. (6)

Section-C

9. 2*8=16

a) Let $f(x,y)=\frac{x-y}{x+y}$, . Show that $\lim_{x\to0}[\lim_{y\to0} f(x,y)] \ne \lim_{(x,y)\to(0,0)} f(x,y)$.

b) If $(x,y)=\cos x\cosh y, g(x,y)=\sin x\sinh y$. Check whether $f(x,y)$ and $g(x,y)$ are functionally related.

c) Find the implicit function defined by the relation $x^2+xy+y^2=7$ near the point (2,1) and find the derivative with respect to x at $x=2$.

d) Discuss the continuity of function $f(x,y)=\begin{cases} xy\left(\frac{x^2-y^2}{x^2+y^2}\right), & (x,y)\ne(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$ at the origin.

e) Evaluate $\int_C (x+y)\,ds$ where C is the line segment $x=t, y=(1-t), z=0$, from $(0,,1,1) to\ (1,0,1)$.

f) Evaluate $\int\int_E e^{x^2+y^2} dxdy$, where E is the region bounded by $y=0$ and the semi circle $y=\sqrt{1-x^2}$.

g) Find the volume of the parabolic cylinder $z=4-x^2$ and the planes $x=0, y=0, y=4, z=0$ assuming the density to be constant.

h) Find the Jacobian $\partial(x,y,z)/\partial(u,v,w)$ of the transformation $x=u\cos v,\ \ y=u\sin v,$ $z=w$.