Time: -3 hrs **M. M. 40**

C-2110 **Algebra-I** 8521/NH

[redacted]

Note: - Candidate are required to Attempt five questions in all selecting not more than two questions from each Section-A and B and Section-C is compulsory.

Section-A **12 marks**

Q1:- (a) Show that Special linear group of degree 'n' denoted by S(L,R) is non-abelian.

(b) Let G be a finite group and let $a \in G$ be an element of order n. Then show that $a^m = e$ if and only if n is a divisor of m.

Q2:- (a) If H is a subgroup of G of index 2 in G. Then prove that H is normal subgroup of G.

(b) State and prove cayley's theorem.

Q3:- If H and K are finite subgroups of a group G, and then prove that

$$o(HK) = \frac{o(H)o(K)}{o(H \cap K)}$$

Q4:- Show that every group of prime order is cyclic.

Section-B **12 marks**

Q5:- A Gaussian integer is a complex number $a + ib$ where a and b are integers. Show that the set J[i] of all Gaussian integers is a ring with usual addition and multiplication of complex numbers.

Q6:- If I and J be any two ideals of a ring R, then prove that IJ is an ideal of R. Moreover $IJ \subseteq I \cap J$.

Q7:- (a) Prove that a division ring is a simple ring.

(b) Find the field of quotients of the integral domain $Z[\sqrt{2}]$.

Q8:- State and prove Fundamental theorem of Ring Homomorphism.

Section-C **16 marks**

Q9:- (a)In a semi group show that cancellation law may not hold.

(b) Define order of an element.

(c) Prove that the centre Z(G) of a group G is a normal subgroup of G.

(d) Prove that every group of Composite order possesses proper subgroups.

(e) Let R and S be two rings. A homomorphism $f: R \to S$ is injective if and only if $Kerf = \{0\}$.

(f) Show that the set $12Z$ is an ideal of the ring $3Z$.

(g) Describe the quotient ring 3Z/12Z.

(h) State third isomorphism theorem on rings.