

E-28/2110

9473/NJ

Mathematical Methods (BHM-303)

(Sem- 3rd)

Note: The question paper consists of NINE questions. The candidates are required to attempt any FOUR questions. All questions carry equal marks.

1. a) State and prove Orthogonal property of Legendre polynomials. (6)
b) Prove the Recurrence relation $(2n+1) P_n(x) = P'_{n+1}(x) - P'_{n-1}(x)$. (6)
c) Expand $f(x) = 3x^2 - 3x + 1$ in terms of Fourier-Legendre series. (5.5)
2. a) Find the generating function for Chebyshev polynomials of first kind. (6)
b) Expand $f(x) = x^3 + x, -1 \leq x \leq 1$ in terms of Chebyshev polynomials of second kind. (6)
c) Prove that $T_n(x) = U_n(x) - x U_{n-1}(x)$. (5.5)
3. a) Prove that $J'_1(x) = J_0(x) - \frac{1}{x} J_1(x)$. (6)
b) Find the solution of the differential equation $xy'' + y' + y = 0$. (6)
c) Find the eigenvalues and eigenfunctions of the boundary value problem $y'' + \lambda y = 0, y(0) = 0, y(l) = 0$, where l is arbitrary. (5.5)
4. a) Find the complete solution of Bessel's equation of order ν , where ν is not an integer. (6)
b) State and prove the Rodrigue's formula. (6)
c) Show that $J_0^2 + 2(J_1^2 + J_2^2 + J_3^2 + \dots) = 1$ (5.5)
5. a) State and prove second shifting theorem of Laplace transform. (5.5)
b) Solve the initial value problem $y'' + 4y' + 4y = 12t^2 e^{-2t}, y(0) = 2, y'(0) = 1$. (6)
c) Find the inverse Laplace transform of $\frac{6+s}{s^2+6s+13}$ (6)
6. a) Solve the initial value problem $y'' + 16y = \delta(t - 2), y(0) = 0, y'(0) = 0$ using convolution theorem. (6)
b) Find Laplace transform of $f(t) = \begin{cases} 5, & 0 \leq t < 2 \\ -5, & t \geq 2 \end{cases}$ (6)
c) Find the inverse Laplace transform of $\frac{1}{(s+5)^4}$ using differentiation of Laplace transform. (5.5)
7. a) Find the Fourier series expansion of $f(x) = x^2; -3 < x < 3$. (5)
b) Find the Fourier series expansion of $f(x) = \pi + x; -\pi < x < \pi$. Hence show that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots \dots \dots \quad (8)$$

c) Find inverse Fourier transform of $\frac{e^{4iw}}{3+iw}$ (4.5)

8. a) Solve $y' - 2y = u_0(t)e^{-2t}$, $-\infty < t < \infty$ using Fourier transform. (8.5)

b) Find the Fourier integral representation of the function (9)

$$f(x) = \begin{cases} -1, & -2 < x < 0 \\ 1, & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

9. a) Prove that Chebyshev polynomial $T_n(x)$ of first kind is a polynomial of degree n , with leading coefficient 2^{n-1} .

b) Show that $P'_n(-x) = (-1)^{n+1}P'_n(x)$.

c) Prove that $xJ'_v(x) = vJ_v(x) - xJ_{v+1}(x)$.

d) Find the Laplace transform of $\sinh t \sin t$.

e) What are the necessary conditions for the convergence of Fourier series of a function.

f) Using convolution theorem, find the Laplace transform of $t^2 e^{3t}$

g) Find Fourier transform of $\frac{1}{(1+t)^2}$ (7*2.5 = 17.5)