

**E-29/2110**

**9480/NJ**

**Linear Integral Equations (BHM-505)  
(Semester V<sup>th</sup>)**

**Max. Marks: 70**

(Attempt any four questions. All questions carry equal marks)

Q1: Solve  $u(x) = 1 + \int_0^x u(t)dt$  using method of successive approximations.

Q2: State and prove Abel's problem.

Q3: Form an integral equation from the given boundary value problem  $\frac{d^2y}{dx^2} + y = \cos x$  with the given conditions  $y(0) = 0$ ,  $y'(0) = 1$  and  $y''(0) = 2$ .

Q4: If  $K(x,t)$  is real and continuous and non-zero in  $R$  and  $f(x)$  is real continuous and non-zero in  $I$ . If there exists a function  $k(x,t)$  reciprocal to  $K(x,t)$  then the Fredholm's equation  $u(x) = f(x) + \int_a^b K(x,t)u(t)dt$  has one and only one continuous solution  $u(x) = f(x) - \int_a^b k(x,t)f(t)dt$

Q5: State and prove Hadamard's theorem.

Q6: Show that  $D(x,y;\lambda) - \lambda K(x,y)D(\lambda) = \lambda \int_a^b K(x,t)D(t,y;\lambda)dt$

Q7: Define symmetric kernel and if  $K(x,t)$  is real, continuous, non-zero and symmetric in  $R$  then  $K_n(x,t) \neq 0$  in  $R$ .

Q8: Show that  $\lambda = 1$  is a characteristic constant of  $K(s_0,s)$  of index 1.

Q9: Let  $C$  be a closed curve with no multiple points and is of class  $c^2$  represented by parameter  $s$  as  $x=\xi(s)$  and  $y=\eta(s)$ . Any line parallel to  $x$ -axis or  $y$ -axis meet the curve almost at  $m$  points where  $m$  is positive integer dividing the plane into interior and exterior region  $C_i$  and  $C_e$ , respectively. A function  $F(s)$  is continuous on  $C$  with  $0 \leq s \leq 1$ , i.e.,  $F(0) = F(1)$ , then  $u(x,y)$  is harmonic on  $I$  s.t.  $u_i(x_0,y_0) = F(s_0)$ , i.e.,  $u(x,y) \rightarrow u_i(x_0,y_0)$  as  $s \rightarrow s_0$ .