

CS/2110

SHEET NO. 1
Extra Copy

5237/NH

Total No. of Sheets used

Total No. of Questions

Subject

Paper

Title of the Paper

Time allowed

Maximum Marks

Minimum Pass Marks

Note : The candidates are required to attempt two questions each from Section A & B Section C will be compulsory

Please assign marks to each question

SECTION A

1(a). Prove that a set containing n distinct elements has 2^n subsets. (3)

1(b). Suppose R is an equivalence relation on a set X. Then

(i) $a \in [a] \forall a \in X$. (ii) $a \in [b]$ if and only if $[a] = [b] \forall a, b \in X$.

(iii) $[a] = [b]$ or $[a] \cap [b] = \emptyset \forall a, b \in X$. (3)

2(a) Suppose that 100 of the 120 students of mathematics at a college take at least one of the languages French, German and Russian. Also suppose 65 study French, 20 study French and German, 45 study German, 25 study French and Russian, 42 study Russian, 15 study German and Russian,

- (i) Find the number of students who study all three languages.
- (ii) Find the number of students who study exactly two languages.
- (iii) Find the number of students who study exactly one language. (3)

2(b) Explain types of Grammars and Languages. (3)

3(a) Show that lattice with three or fewer elements is a chain. (3)

3(b) A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has

- (i) No girl
- (ii) At least one boy and one girl
- (iii) At least three girls. (3)

4(a) If A, B, C are three events, then prove that

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$ (3)

4(b) Prove by mathematical induction that if $A_1, A_2, A_3, \dots, A_n$ are any n sets then

$\overline{(\bigcap_{i=1}^n A_i)} = \bigcup_{i=1}^n \bar{A}_i$ for $n \geq 1$ (3)

SECTION-B

5(a) State and prove Euler's Formula for planar graph. (3)

5(b) Show that $K_{3,3}$ satisfies the inequality $|E| \leq 3|V| - 6$, but is not planar. (3)

6(a) Discuss Konigsberg's graph. (3)

Contd. ...

6(b) Prove that a graph is connected if and only if it has a spanning tree. (3)

7(a) A three-state finite state machine has $\{0, 1\}$ as its input and output alphabets. Given the following input sequence and its corresponding output sequence, determine the machine.

Input sequence: 00010101

Output sequence: 011001110

(3)

7(b) Write a short note on Finite State Machines. (3)

8(a) Give an example of graph that has

(i) Euler circuit but not Hamiltonian circuit.

(ii) Hamiltonian circuit but not Euler circuit.

(3)

8(b) Prove that the number of edges in a complete graph with n vertices is $\frac{n(n-1)}{2}$. (3)

SECTION-C

9(a) Use Pigeonhole Principle to find how many people among 200000 people are born at same time (hour, minute, second)?

9(b) Define Partial order relation with example.

9(c) What is the chance that a leap year selected at random will contain 53 Sundays.

9(d) How many different signals can be formed with five given flags of different colors?

9(e) Discuss Travelling Sales Person Problem using graph theory.

9(f) Show that there is one and only one path between every pair of vertices in a tree.

9(g) Explain The Breadth First Search Algorithm to find the shortest path between two vertices in a graph without weights.

9(h) Define weighted graph.

$2 \times 8 = 16$

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