Time: -3 hrs CS/2110 5238/NH

M. M. 40

Mathematical Methods-I(opt.-1)

[redacted]-5th Sem

Note: - Candidate are required to Attempt five questions in all selecting not more than two questions from each Section-A and B and Section-C is compulsory.

Section-A **12 marks**

Q1:- Find the Fourier series to represent $f(x) = xsinx$ from $x = 0 \; to \; x = 2\pi$.

Q2:- Expand in a series of sines and cosines of multiple of x, the function given by

$f(x) = \begin{cases} \pi + x \; when - \pi < x < 0 \\ \pi - x \; when \; 0 < x < \pi \end{cases}$. What is the sum of the series for $x = \pm\pi \; and \; x = 0$.

Q3:- Expand $f(x) = x^2$ as Fourier series in interval $-2 \leq x \leq 2$.

Q4:- If $(x) = \begin{cases} x \; when \;\; 0 < x < \pi/2 \\ \pi - x \; when \; \pi/2 < x < \pi \end{cases}$, Show that $f(x) = \frac{\pi}{4} - \frac{2}{\pi}\left[cos2x - \frac{cos6x}{3^2} + \frac{cos10x}{5^2} + \cdots\right]$.

Section-B **12 marks**

Q5:- State and prove Existence theorem of Laplace transform.

Q6:- (a) Find the Laplace transform of $L\{sinh^3 2t\}$

(b) State and prove first shifting theorem.

Q7:- (a) Evaluate: $L^{-1}\left(\frac{2s}{s^4+s^2+1}\right)$.

(b) Find the inverse Laplace transform of $\frac{1}{\sqrt{2s+3}}$.

Q8:- State and prove Heaviside's Expansion Formula. Using Heaviside's expansion formula, find $L^{-1}\left(\frac{s^2-10s+13}{(s-7)(s^2+5s+6)}\right)$.

Section-C **16 marks**

Q9: (a) State Dirichlet conditions.

(b) If m and n are the integers, then evaluate $\int_a^{a+2\pi} \sin nx \cos mx \, dx$.

(c) Foe a periodic function of period 2π, prove that $\int_{-\pi}^{\pi} f(x)\, dx = \int_{-\pi}^{\pi} f(c+x)\, dx$.

(d) State Riemann-Lebesgue theorem.

(e) Prove that $f(x) = x^n$ is of exponential order $\alpha > 0 \; as \; x \to \infty$, where $n \in N$.

(f) Prove that for $t \geq 0, L(1) = \frac{1}{s}$.

(g) State Convolution theorem.

(h) Prove $\int_0^{\infty} \frac{sint}{t} dt = \frac{\pi}{2}$.