compulsory

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B-2110 8462/NH

ANALYSIS-I

Paper-II

(Semester-III)

Time: Three Hours Maximum Marks: 40

Note: Attempt five questions in all. Select two questions each from Section A and B while Section C is compulsory.

SECTION-A

I. State and prove Cauchy's First Theorem on Limits. 6

II. (a) Prove that a convergent sequence is always a Cauchy sequence. 3

(b) If $\{a_n\}$, $\{b_n\}$ and $\{c_n\}$ are three sequences such that

$b_n \leq a_n \leq c_n \ \forall \ n \in \mathbb{N}$ and $b_n \to l$, $c_n \to l$, then $a_n \to l$. 3

III. (a) If $\{a_n\}$ is monotonic decreasing sequence of positive terms and converges to zero, then $\sum_{n=1}^{\infty}(-1)^{n-1}a_n$ is convergent. 4

(b) Discuss the convergence or divergence of the series $\sum \frac{1}{n} \tan \frac{1}{n}$. 2

IV. (a) If $\sum a_n$ is a positive terms convergent series, then prove that $\sum a_n^2$ is convergent. Is its converse true? 4

(b) Discuss the convergence or divergence of the series $\sum\left(\sqrt[3]{n^3+1}-n\right)$. 2

SECTION-B

V. A function f is R-integrable on [a, b] and there exists a function F such that F'=f on [a, b],then show that $\int_a^b f\,dx = F(b) - F(a)$. 6

Contd. - -

VI. (a) Prove that every monotonic function defined on [a, b] is Riemann integrable. 3

(b) Given an example of bounded function which is not R-integrable over [0,1]. 3

VII. (a) Compute $\int_0^5 [x]dx$,where [x] denotes the greatest integer not greater than x. 3

(b) Let a function F is R-integrable on [a, b], then show that the function F defined as

$F(x) = \int_a^x f(t)dt$,a$\leq x \leq b$ is continuous on [a, b]. 3

VIII. (a) For a function F, V(f,[a, b])=0 iff F is a constant on [a, b]. 3

(b) Let F be continuous on [a, b]. Then F is of bounded variation on [a, b] iff F can be expressed as difference of two increasing continuous functions. 3

SECTION-C

IX. (a) Define convergent sequence.

(b) Prove that $\lim_{n\to\infty} \frac{1}{n}\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right) = 0$

(c) Is $\sum_{n=1}^{\infty} \frac{n^2-1}{n^2+1}$ convergent? justify your answer.

(d) Show that the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$ is convergent.

(e) Show that the function f defined by

f(x)=$\begin{cases} 0, when\ x\ is\ rational \\ 1, when\ x\ is\ irrational \end{cases}$ Is not integrable on any interval.

(f) IF f and g are integrable on [a, b] and f(x) ≤ g(x) ∀ x ∈ [a, b], then show that $\int_a^b f(x)dx \leq \int_a^b g(x)dx$.

(g) Define Total Variation of function.

(h) If f is increasing function on [a, b], then prove that f is function of bounded variation on [a,b]. 2 x 8 = 16

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