C-2110

Time: -3 hrs M. M. 40

Mathematical Methods-I(Opt.-1[redacted] 8523/NH

Note: - Candidates are required to Attempt five questions in all selecting not more than two questions from each Section-A and B and Section-C is compulsory.

Section-A 12 marks

Q1:- Find the Fourier series to represent $x - x^2$ from $x = -\pi$ to $x = \pi$. Hence show that

$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12}$.

Q2:- Expand in a series of sines and cosines of multiple of x, the function given by

$f(x) = \begin{cases} x - \pi \text{ when } -\pi < x < 0 \\ \pi - x \text{ when } 0 < x < \pi \end{cases}$. What is the sum of the series for $x = \pm\pi$ and $x = 0$.

Q3:- Expand $f(x) = e^{-x}$ as Fourier series in interval $-l < x < l$.

Q4:- If $(x) = \begin{cases} x \text{ when } 0 < x < \pi/2 \\ \pi - x \text{ when } \pi/2 < x < \pi \end{cases}$, Show that

(i) $f(x) = \frac{4}{\pi}[\sin x - \frac{\sin 3x}{3^2} + \frac{\sin 3x}{3^2} + \cdots]$.

(ii) $f(x) = \frac{\pi}{4} - \frac{2}{\pi}[\cos 2x - \frac{\cos 6x}{3^2} + \frac{\cos 10x}{5^2} + \cdots]$

Section-B 12 marks

Q5:- State and prove Existence theorem of Laplace transform.

Q6:- (a) Find the Laplace transform of $\cos\sqrt{t}$.

(b) State and prove second shifting theorem.

Q7:- (a) Evaluate: $L^{-1}\left(\frac{s^2+s-2}{s(s+3)(s-2)}\right)$.

(b) Find the inverse Laplace transform of $\frac{2}{s^5+4s^3}$.

Q8:- State and prove Heaviside's Expansion Formula. Using Heaviside's expansion formula, find $L^{-1}\left(\frac{6s^2+22s+18}{s^3+6s^2+11s+6}\right)$.

Section-C 16 marks

Q9: (a) Define Fourier series.

(b) If m and n are the integers, then evaluate $\int_a^{a+2\pi} \sin nx\, dx$.

(c) Foe a periodic function of period 2π, prove that $\int_a^b f(x)\,dx = \int_a^{b+2\pi} f(x)\,dx$.

(d) State Riemann-Lebesgue theorem.

(e) Define Beta function.

(f) Prove that for $t \geq 0, L(1) = \frac{1}{s}$.

(g) State Change of scale property.

(h) Prove $\int_0^\infty \frac{\sin t}{t}\,dt = \frac{\pi}{2}$.