F-40/2110

MATHEMATICAL METHODS-303

SEMESTER-III

TIME ALLOWED 3 Hrs M.M 70

NOTE: The candidates are required to attempt two questions each from Section A & B Section C will be compulsory.

Section A

1. Expand $f(x) = x^3 + x, -1 \le x \le 1$, in terms of Chebyshev series of first kind $T_n(x)$.
2. (a) State and prove Rodrigue's formula.
   (b) Prove that $T_{2n+1}(x) = 0$
3. (a) Prove that $J_1''(x) = \frac{1}{x}J_2(x) - J_1(x)$.
   (b) Using the recurrence relation $(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$ recursively. Evaluate $P_3(2.1)$.
4. Find the Eigen functions of the following Sturm-Liouville problems and verify their orthogonality $\frac{d^2y}{dx^2} + \lambda y = 0,\ y'(0) = 0,\ y(\pi) = 0$

2×10=20

SECTION B

5. (a) Show that $\int_0^\infty te^{-3t}\sin t\ dt = \frac{3}{50}$ by using the Laplace transformation. **(5)**

   (b) Find the Laplace transform of the function $f(t) = \begin{cases} \sin\omega t & ; 0 < t < \pi/\omega \\ 0 & ; \pi/\omega < t < 2\pi/\omega \end{cases}$. **(5)**

6. (a) Find the inverse Laplace transform of the function $\frac{s}{(s^2+a^2)^3}$. **(5)**

   (b) Show $1*1*1*\ldots\ldots*1(n \text{ times}) = \frac{t^{n-1}}{(n-1)!}$ where $n = 1,2,3,\ldots$. **(5)**

7. Write the Fourier series expansion for $f(x)$ if $f(x) = \begin{cases} -\pi & ; -\pi < x < 0 \\ x & ; 0 < x < \pi \end{cases}$ and hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots = \frac{\pi^2}{8}$.
8. Solve the initial value problem $\frac{d^2y}{dx^2} + 9y = 0\ ;\ y(0) = 0,\ y'(0) = 1$ with the help of Laplace transforms.

2×10=20

SECTION C

9. Write in brief.
   a) Find $L\left[\frac{\sin at}{t}\right]$, given that $L\left[\frac{\sin t}{t}\right] = \tan^{-1}\frac{1}{s}$.
   b) State Bessel functions of first kind $J_n(x)$ and second kind $Y_n(x)$.
   c) Find the Laplace transform of $e^t\cos t$.
   d) Write Euler's formula for Fourier series.

e) State Convolution theorem.

f) State the necessary and sufficient conditions of Laplace Transform.

g) Find the inverse Laplace transform of $\frac{8(s+2)}{(s^2+4s+8)^2}$

h) State the orthogonal property of Legendre polynomial and Chebyshev polynomial of second kind.

i) Define Generating function of the Legendre polynomial.

j) Find the Laplace transformation of $e^{-t}\int_0^t \frac{\sin t}{t}\,dt$.

10X3 = 30