F-57/2110 10321/NJ

Discrete Mathematics (CSM-353)
(Semester-5TH)
(Syll-Dec-2019)

Attempt any four questions. All questions carry equal marks.

Section-A

1. (i) Prove that for any positive integer n $(n \geq 2)$, it is either a prime or product of primes.
   (ii) Find how many integers between 1 and 60 are neither divisible by 2, nor by 3 and nor by 5.
2. (i) Prove that the set of natural numbers has same cardinality as the set of even integers.
   (ii) State and prove the first theorem on graph theory.
3. (i) Let R be an equivalence relation on a set A. Then, any two equivalence classes are either disjoint or identical.
   (ii) Prove that the relation of inclusion on $P(A)$ $(A \neq \emptyset)$ is a partial order relation.
   (iii) Define absolute value function with suitable example.
4. (i) State and prove Euler's formula for a connected planar graph.
   (ii) Define generating function with suitable example.

Section-B

5. (i) Using the generating function, solve the recurrence relation

   $$S_n - 2S_{n-1} - 3S_{n-2} = 0, n \geq 2, S_0 = 3, S_1 = 1.$$

   (ii) Define Fibonacci sequence recursively.
6. (i) Prove that $(D_{30}, \leq)$ where $\leq$ denotes the relation of divisibility, is a lattice.
   (ii) Define complemented lattice with suitable example.
7. (i) State and prove De-Morgan's laws of Boolean algebra.
   (ii) What are atoms and anti-atoms in a Boolean algebra.
8. (i) Check the validity of following argument: If a man is bachelor, he is unhappy. If a man is unhappy, he dies young. Bachelors die young.
   (ii) State the converse, inverse and contrapositive of followingimplications :

a. If 4x-2=10, then x=3.
b. If it snows tonight, then I will stay at home.

Section-C

9. (i) Define partitioning of sets. Illustrate with the help of a venn diagram.
(ii) Show that if 9 colors are used to paint 1000 houses, atleast 112 houses will be of the same colour.
(iii) What is XOR gate.
(iv) DrawHasses diagram of D_{36} under the relation of divisibility.
(v) Discuss in brief the concept of switching circuits.