Roll No. ..... Total Pages : 5

10371/NH

AS/2110

CO-ORDINATE GEOMETRY

Paper-III

Semester-I

Syllabus-(Dec-19)

Time allowed : 3 Hours] [Maximum Marks : 40

Note : The candidates are required to attempt two questions from Section A and B carrying 6 marks each and the entire Section C consisting of 8 short answer type questions carrying 2 marks each.

SECTION-A

1. (i) If the chord of the parabola $y^2 = 4ax$ subtends a right angle at the vertex of the parabola, show that the tangents at its extremities meet in the same line.

(ii) Prove that the circle described on any focal radius of a parabola as diameter touches the tangent at the vertex.

2. (i) Prove that the semi-latus rectum of a parabola is the harmonic means between the segments of a focal chord.

(ii) State and prove the diameter property of $y^2 = 4ax$.

3. (i) Prove that the locus of the poles of tangents to the parabola $y^2 = 4bx$ is the parabola

$$y^2 = \frac{4b^2}{a} x$$

(ii) Find the vertex, focus, axis, directrix and length of the latus rectum

$x^2 + 8x + 12y + 4 = 0$.

4. (i) When is the line $lx - my + n = 0$ is :

(a) tangent

(b) normal to the parabola $y^2 = 4ax$?

(ii) Prove that the subnormal at any point of a parabola is constant and equals half of its latus rectum.

SECTION-B

5. (i) Find the equation of tangent and normal to the ellipse $x^2 + 5y^2 = 14$ at (3, 1).

(ii) Show that the joint equation of pair of tangents drawn to the ellipse

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

from the point (4, 3) is $xy - 3x - 4y - 12 = 0$.

Also prove that they are at right angles.

6. (i) Find the pole of the chord $x - 2y = 1$ w.r.t. the hyperbola $4x^2 + 9y^2 = 12$.

(ii) Find the point of intersection of the line $y = mx + c$ with the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Also deduce the condition of tangency.

7. (i) Find the equations of directrices, centre, foci of the hyperbola $16x^2 - 9y^2 - 32x + 36y - 164 = 0$.

(ii) Prove that the line $y = mx + c$ is a normal to

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ if } \frac{a^2}{m^2} + b^2 = \frac{(a^2 - b^2)^2}{c^2}$$

8. (i) State and prove reflexive property of the ellipse.

(ii) A tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ cut the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in P and Q. Find the locus of middle point of [PQ].

SECTION-C

9. (i) Define Auxiliary circle of the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$, what is its equation ?

(ii) Under what condition that pair of lines $Ax^2 + 2Hxy + By^2 = 0$ may be the conjugate diameters of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

(iii) Is the line $x - 2y + 3 = 0$ tangent to $y^2 = 16x$.

(iv) Find the co-ordinates of the point where the line $5x = 4y$ meets the curve

$$\frac{x^2}{16} + \frac{y^2}{25} = 1$$

Also find length of chord intercepted.

(v) What are the parameteric representation of any point on the parabola $y^2 = 4ax$.

(vi) Under what condition $x \cos \quad + y \sin \quad = p$ is normal to $2x^2 + 3y^2 = 1$.

(vii) If e and e' are the eccentricities of hyperbola and its conjugate hyperbola then

$$e' = \frac{e}{\sqrt{e^2 - 1}}$$

(viii) Write down the any two properties of :

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

8×2=16