M-39/2110 10491/N

DIFFERENTIABLE MANIFOLDS (MM-601/ AMC-308)
(SEMESTER-3RD)

Time allowed:2hours Max. Marks:70

Note: Attempt four questions in all carrying 17.5 marks each.

Q.1. Discuss the concept of a tangent vector as a directional derivative operator. Hence prove that the dimension of a tangent space at a point of a manifold equals the dimension of themanifold.

Q.2. Define the Lie-derivative of a tensor field. Prove that the Lie-derivative and exterior derivative commute.

Q.3. Explainthelocalcoordinateapproachtoadifferentiablemanifoldofdimension$n$.Henceshowthat a two-dimensional sphere is an orientablemanifold.

Q.4. Discuss the theory of induced metric in a submanifold of a Riemannian manifold. Also prove that the second fundamental form is symmetric and bilinear on $C^\infty(M)$.

Q.5. Discuss the concept of wedge product. State and prove any two properties of the wedge product of contravarianttensors.

Q.6. From the local coordinate approach to curvature tensor of a manifold, express the components of curvature tensor in terms of connectioncoefficients.

Q.7. Define a Levi Civita connection. Hence prove the fundamental theorem of Riemanniangeometry.

Q.8. State and prove Bianchi's first identity for an affine connection of amanifold.

Q.9.(a)For$f \in C^\infty(M)$andforany$r$-form$w$on$M$,provethat$d(fw) = df \wedge w + fdw$.

(b)DerivetheChristoffelsymbolsofthefirstandthesecondkind.Also,stateandprovetherelationship between thetwo.

(c) WhatisaDifferencetensor?Provethatthetwoconnectionsareequaliffthéyhavesamegeodesics and same torsion tensors.

(d) Prove that Jacobian preservesLie-bracket.

(e) Defineanexterior$r$-vectorandan$r$-form.Computetheexteriorproduct$(6du^1 \wedge du^2 + 27du^1 \wedge du^3) \wedge (du^1 + du^2 + du^3)$.

(f) Define a sectional curvature and a spaceform.

(g) UsingGaussandWeingartenformulae,provethat $g(A_N X, Y) = g(h(X,Y), N)$.

(h) Prove Bianchi's first identity for the Riemannian curvaturetensor.

(i) Show that an affine connection can be decomposed into a sum of a multiple of its torsion tensor and a torsion freeconnection.

(j) When are the coordinate charts said to be C^∞compatible?