M-39/2110 10499/N

Optimization Techniques I-MM 609/AMC-316
(Semester III)
(Common for Math/AMC)
M.Sc Math/M.Sc. Maths (AMC))
(Syll-Dec-2019)

Time: 3 Hours Maximum marks: 70

Note: Attempt any four questions. All Questions carry equal marks

I (a) Solve graphically the following LPP

$\text{Max.} z = 20x_1 + 10x_2$

$\text{Sub. to } x_1 + 2x_2 \le 40,\ 3x_1 + x_2 \ge 30,\ 4x_1 + 3x_2 \ge 60$

$x_1,\ x_2 \ge 0$

(b) Solve the L.P.P. using Big M method:

Maximize $z = x_1 + 2x_2 + 3x_3 - x_4$

subject to constraints:

$x_1 + 2x_2 + 3x_3 = 15,\ 2x_1 + x_2 + 5x_3 = 20,\ x_1 + 2x_2 + x_3 + x_4 = 10,$

$x_1, x_2, x_3, x_4 \ge 0$

II (a) State and prove complimentary slackness theorem.

(b) Use Branch and Bound method to solve the following Integer Linear programming problem:

Maximize $z = 2x_1 + 3x_2$

subject to constraints:

$5x_1 + 7x_2 \le 35, 4x_1 + 9x_2 \le 36,\ \ x_1, x_2 \ge 0$ are integers

III (a) Solve the following mixed integer programming problem.

Maximize $z = 4x_1 + 6x_2 + 2x_3$ subject to constraints

$4x_1 - 4x_2 \le 5,\ -x_1 + 6x_2 \le 5,\ -x_1 + x_2 + x_3 \le 5$

$x_1, x_2, x_3 \ge 0;\ x_1$ and x_3 are integers

(b) The manager of an oil refinery must decide on the optimum mix of two possible blending processes of which the input and output production runs are as follows:

Process	Input		Output	
	Crude A	Crude B	Gasoline X	Gasoline Y
1	6	4	6	9
2	5	6	5	5

The maximum amounts available of crudes A and B are 250 units and 200 units respectively. Market demand shows that at least 150 units of gasoline X and 130 units

of gasoline Y must be produced. The profits per production run from process 1 and process 2 are Rs. 4 and Rs.5 respectively. Formulate the problem for maximizing the profit.

IV Given the LPP

Maximize $z = 3x_1 - 2x_2 + 4x_3$ subject to constraints:
$x_1 + 2x_2 + x_3 \leq 430, 3x_1 + 2x_3 \leq 460, x_1 + 4x_2 \leq 420$
$x_1, x_2, x_3 \geq 0$

BV	x_1	x_2	x_3	s_1	s_2	s_3	Sol
z	3	2	0	0	2	0	920
s_1	-1/2	2	0	1	-1/2	0	200
x_3	3/2	0	1	0	1/2	0	230
s_3	1	4	0	0	0	1	420

If we add $x_1 + x_3 \leq 200$ to this LPP, find the new optimal solution of the modified problem using sensitivity analysis.

V A product is manufactured by four factories A, B, C and D. The unit production costs in them are Rs. 2, Rs.3, Rs.1 and Rs.5 respectively. Their production capacities are 50, 70, 30 and 50 units respectively. These factories supply the product to four stores, demands of which are 25, 35, 105 and 20 units respectively. Unit transportation cost in rupees from each factory to each store is given in the table below:

		Stores			
		1	2	3	4
Factories	A	2	4	6	11
	B	10	8	7	5
	C	13	3	9	12
	D	4	6	8	3

Determine the transportation plan to minimize the total production –cum-transportation cost

VI Solve the following assignment problem of minimizing total time of doing all the Jobs.

	JOBS				
	I	II	III	IV	V
1	6	2	5	2	6
2	2	5	8	7	7

3	7	8	6	9	8
4	6	2	3	4	5
5	9	3	8	9	7
6	4	7	4	6	8

VII Solve the following game by linear programming technique:

$$\begin{array}{cc} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & -1 & 3 \\ 3 & 5 & -3 \\ 6 & 2 & -2 \end{bmatrix} \end{array} \quad \begin{matrix} \\ \\ player\ A \end{matrix} \quad player\ B$$

VIII (a) Find the optimal strategies for both-persons and the value of game for zero-sum two-persons game graphically, whose payoff matrix is as follows:

$$\begin{bmatrix} 2 & 1 & 0 & -2 \\ 1 & 0 & 3 & 2 \end{bmatrix}$$

(b) Solve the following game using dominance property

$$\begin{array}{cc} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 10 & 5 & -2 \\ 13 & 12 & 15 \\ 16 & 14 & 10 \end{bmatrix} \end{array} \quad \begin{matrix} \\ \\ player\ A \end{matrix} \quad player\ B$$

IX (a) Prove that Dual of a Dual is primal.

(b) What are the limitations of O.R. techniques

(c) Write a short note on post optimal analysis.

(d) Explain the difference between transportation problem and assignment problem

(e) When the problem of degeneracy arises in transportation problem? How do we handle this problem?