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10662/NH

C/2110

ADVANCED CALCULUS

Paper-I

Semester-III

Syllabus-(Dec-19)

Time allowed : 3 Hours] [Maximum Marks : 40

Note : The candidates are required to attempt two questions each from Section A and B. Section C will be compulsory.

SECTION-A

1. (i) Show that the function

$$f(x, y) = \frac{xy^3}{x^2 + y^6}, \quad x \quad 0, \; y \quad 0 \text{ and } f(0, 0) = 0$$

is not continuous at (0, 0) in (x, y) together but the function is continuous in x-alone and y-alone at the origin.

(ii) Using – definition, prove that :

$$\lim_{(x, y) \to (2, 1)} (3x - 4y) = 2$$

2. (i) If $u = xf(y/x) + y\ (y/x)$, prove that :

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \, \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 0$$

(ii) State and prove Schwarz's theorem.

3. (i) If $u = x, y, z ; v = xy + yz + zx , w = x + y + z,$

Show that :

$$\frac{(x, y, z)}{(u, v, w)} = \frac{1}{(x - y)(y - z)(z - x)}$$

(ii) Compute the value of cos 32° correct to four decimal places using Taylor's series.

4. (i) Using Lagrange's method of multipliers, find the point on the plane $2x - 3y + 6z = 49$ nearest to the origin of IR^3.

(ii) Find the possible percentage error in computing the resistance r from the formula

$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2}$ if r_1 and r_2 both in error by 2%.

SECTION-B

5. (i) Evaluate :

$$\int_0^{\infty}\int_x^{\infty} \frac{e^{-y}}{y}\, dy\, dx$$

(ii) Transform $\int_0^{\infty}\int_x^{a} r^3 \sin \cos \, dr d$ to Cartesian form and evaluate.

6. (i) Find the smaller of the areas bounded by the ellipse $4x^2 + 9y^2 = 36$ and the straight line $2x + 3y = 6$.

(ii) Calculate, by double integration, the volume generated by the revolution of the cardioid $r = a(1 - \cos\ )$ about its axis.

7. Find the C.G. of the area between $y = 6x - x^2$ and $y = x$ (C.G. = Centre of gravity)

8. (i) Find, by double integration, the area enclosed by the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

(ii) Evaluate.

$$\int_0^{\infty}\int_0^{a\cos}\int_0^{\sqrt{a^2 - r^2}} r\, dz\, dr\, d$$

SECTION-C

9. (i) Show that $f(x, y) = \sqrt{|xy|}$ is not differentiable at (0, 0).

(ii) If $x = r\cos\ , y = r\sin\ ,$ prove that :

$$\frac{\partial^2}{\partial x\, \partial y} = \frac{\partial^2}{\partial x^2} (\log r)$$

(iii) State Euler's theorem for homogeneous functions and Young's theorem for equality of mixed partial derivatives.

(iv) Expand in powers of $(x - 1)$ and y the function $\sqrt{1 + x + y^2}$.

(v) If $x + y = u$ and $y = uv$, evaluate $\frac{(x, y)}{(u, v)}$.

(vi) If $u = xe^{y}z$ where $y = \sqrt{a^2 - x^2}$, $z = \sin^2 x$, find $\frac{du}{dx}$.

(vii) State necessary condition for a function f(x, y) to have an extremum at (a, b).

(viii) Examine the function $x^3y^2(1 - x - y)$ for saddle point.

(ix) Define moment of inertia and state formulae to calculate moment of inertia of a plane lamina.

(x) Evaluate $\iint_R (x + y)^2 dxdy$, where R is the parallelogram in the xy-plane with vertices (1, 0), (3, 1), (2, 2) and (0, 1).