M-73/2/10 10849/N

MATHEMATICAL STATISTICS
MM 406

Max. Marks: 70

Time allowed: 3 hours

Note: Attempt five questions in all selecting two questions each from Section A and B carrying 10 marks each and the compulsory Section C consisting of ten short answer type questions having 3 marks each.

SECTION-A

Q.1. (a) Discuss the technique of transformation of a function of two dimensional random variable using the Jacobian of the transformation. Hence find the disribution of $X+Y$ if X and Y are independent Gamma variates with parameters μ and ν respectively.

(b) Explain some random phenomena where we apply Poisson distribution. Also show that Poisson distribution is a special case of Binomial distribution. **(6+4)**

Q.2.(a) Define the expectation of a function of a random variable. Show that if X and Y are independent and $g_1(X)$ and $g_2(Y)$ are any two functions of X and Y respectively , then

$$E[g_1(X).g_2(Y)] = E[g_1(X)].E[g_2(Y))]$$

(b)Explain the situation under which a random variable follows Binomial distribution. Find the moment generating function of the Binomial variate and hence find its mean and variance . **(5+5)**

Q.3. Discuss the concept of area under the Normal probability curve. In a Normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution. **(10)**

Q.4. (a) Name two discrete distributions and two continuous distributions where the corresponding random variable is referred to as the waiting time - random variable. Find the mean and the moment generating function in each case.

(b) Discuss the moment generating function technique to determine the distribution of functions of a two random variables. If X_1 and X_2 are two independent standard normal variates, then find the distribution of $Y = \frac{(X_2-X_1)^2}{2}$. **(5+5)**

SECTION-B

Q.5. (a) If Z and U are two independent random variables such that Z has standard normal distribution and U has a Chi-square distribution with k degrees of freedom, then prove that the random variable $\frac{Z}{\sqrt{U\backslash k}}$ has Student's t-distribution with k degrees of freedom.

(b) Show that the critical values of the left-tailed F-test are given by the reciprocal of the critical values of the right-tailed F-test with degrees of freedom reversed. **(6+4)**

Q.6. Discuss the test of significance for difference of proportions, defining the null hypothsis under consideration, the test statistic and the conclusion.
In two large populations, there are 30% and 25% respectively of fair haired people. Is this difference likely to be hidden in samples of 1200 and 900 respectively from the two populations? **(10)**

Q.7. Define a Chi-square variate with n degrees of freedom. Show that if X and Y are two independent Chi- square variates with m and n degrees of freedom, then $\frac{X}{Y}$ is a $\beta_2(\frac{m}{2}, \frac{n}{2})$- variate. **(10)**

Q.8. (a) In the large sample theory, discuss the test of significance of a single mean.

Contd. ...

(b) It is claimed that a random sample of 100 tyres tyres with a mean life of 15269 kms is drawn from a population of tyres which has a mean life of 15200 kms and a standard deviation of 1248 kms. Test the validity of the claim at 5% level of significance. **(4+6)**

SECTION-C

Q.9. (a) Define a sigma-field generated by a non-empty class of subsets and the probability space.

(b) In a sampling from Normal population with mean μ and variance σ^2 , show that sample mean is a consistent estimator of population mean.

(c) Define a random sample and distribution of sample.

(d) A sample of 400 items has a mean of 82 and standard deviation of 18. Find the 95% confidence limits for the mean of the population from which the sample has been taken.

(e) Define a standard Normal variate. Find its m.g.f. and hence derive the mean and variance.

(f) Define a MP test and UMP test for testing of hypothesis.

(g) Find first two central moments of Beta distribution of first kind.

(h) Does additive property hold for the Binomial distribution? Discuss.

(i) Which distribution is a particular case of the Negative Binomial distribution ? Explain by defining their density functions.

(j) If X is a normally distributed with mean 2 and variance 1, then find $P[|X-2|<1]$. **(10×3=30)**

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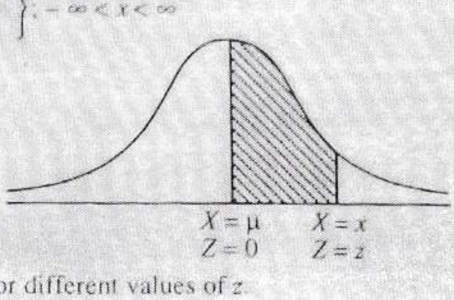
TABLE VI : AREAS UNDER STANDARD NORMAL PROBABILITY CURVE

Normal Probability curve is given by : $f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}; -\infty < x < \infty$

and standard normal probability curve is given by :

$$f(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}z^2\right), -\infty < z < \infty$$



where $$Z = \frac{X - E(X)}{\sigma_x} = \frac{X - \mu}{\sigma} \sim N(0, 1)$$

The following table gives the shaded area in the diagram, *viz.* $P(0 < Z < z)$ for different values of z.

AREAS UNDER STANDARD NORMAL PROBABILITY CURVE

z \ z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

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TABLE VIII : CRITICAL VALUES OF STUDENT'S t-DISTRIBUTION

d.f.	LEVEL OF SIGNIFICANCE FOR ONE-TAILED TEST						
	.25	.10	.05	.025	.01	.005	.0005
	LEVEL OF SIGNIFICNCE FOR TWO-TAILED TEST						
υ	.50	.20	.10	.05	.02	.01	.001
1	1.000	3.078	6.314	12.706	31.821	63.657	636.619
2	.816	1.886	2.920	4.303	6.965	9.925	31.599
3	.765	1.638	2.353	3.182	4.541	5.841	12.924
4	.741	1.533	2.132	2.776	3.747	4.604	8.610
5	.727	1.476	2.015	2.571	3.365	4.032	6.869
6	.718	1.440	1.943	2.447	3.143	3.707	5.959
7	.711	1.415	1.895	2.365	2.998	3.499	5.408
8	.706	1.397	1.860	2.306	2.896	3.355	5.041
9	.703	1.383	1.833	2.262	2.821	3.250	4.781
10	.700	1.372	1.812	2.228	2.764	3.169	4.587
11	.697	1.363	1.796	2.201	2.718	3.106	4.437
12	.695	1.356	1.782	2.179	2.681	3.055	4.318
13	.694	1.350	1.771	2.160	2.650	3.012	4.221
14	.692	1.345	1.761	2.145	2.624	2.977	4.140
15	.691	1.341	1.753	2.131	2.602	2.947	4.073
16	.690	1.337	1.746	2.120	2.583	2.921	4.015
17	.689	1.333	1.740	2.110	2.567	2.898	3.965
18	.688	1.330	1.734	2.101	2.552	2.878	3.922
19	.688	1.328	1.729	2.093	2.539	2.861	3.883
20	.687	1.325	1.725	2.086	2.528	2.845	3.850
21	.686	1.323	1.721	2.080	2.518	2.831	3.819
22	.686	1.321	1.717	2.074	2.508	2.819	3.792
23	.685	1.319	1.714	2.069	2.500	2.807	3.768
24	.865	1.318	1.711	2.064	2.492	2.797	3.745
25	.684	1.316	1.708	2.060	2.485	2.787	3.725
26	.684	1.315	1.706	2.056	2.479	2.779	3.707
27	.684	1.314	1.703	2.052	2.473	2.771	3.690
28	.683	1.313	1.701	2.048	2.467	2.763	3.674
29	.683	1.311	1.699	2.045	2.462	2.756	3.659
30	.683	1.310	1.697	2.042	2.457	2.750	3.646
40	.681	1.303	1.684	2.021	2.423	2.704	3.551
60	.679	1.296	1.671	2.000	2.390	2.660	3.460
120	.677	1.289	1.658	1.980	2.358	2.617	3.373
∞	.674	1.282	1.645	1.960	2.326	2.576	3.291