5196/NH

A-S-2110

LINER ALGEBRA-III
SEMESTER-I

TIME 3 HOURS MM: 40

Note :The candidates are required to attempt two questions each from Section A & B , Section C will be compulsory.

SECTION-A

I State Cayley-Hamilton theorem, verify this theorem for the matrix $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ (6)

II (a) If A is an $\times$ n $matrix$ and $\rho(A) = n-1,$ $show$ $that$ $\rho(adj.A) = 1$ (3)

II (b) Find the values of a such that the rank of the matrix $A = \begin{bmatrix} 3a-8 & 3 & 3 \\ 3 & 3a-8 & 3 \\ 3 & 3 & 3a-8 \end{bmatrix}$ is ≤ 2

Also find the rank for these values of a. (3)

III (a) Find the values of λ and μ so that the system of equations

$x + y + z = 6,\ x + 2y + 3z = 10,\ x + 2y + \lambda z = \mu$

has (i) no solution (ii) a unique solution (iii) an infinite number of solutions (3)

III (b) If equations $x = cy + bz,\ y = az + cx,\ z = bx + ay$ have a non-trivial solution,

then show that the solutions are given by $x = \lambda\sqrt{1-a^2},\ y = \lambda\sqrt{1-b^2},\ z = \lambda\sqrt{1-c^2}$,

where λ is any real number. (3)

IV (a) Diagonalize the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$,if possible. (3)

IV (b) Using Gauss Jordan Method, find the inverse of matrix $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ (3)

SECTION-B

V Prove that there exist a basis for each finite dimensional vector space. (6)

VI Let V be the set of all $n \times n$ skew-symmetric matrices over field **R.** Let vector addition and scalar Multiplication be defined as usual addition of matrices and multiplication of a scalar with a matrix. Show that V is a vector space over **R** (6)

VII (a) Let V be a finite dimensional vector space over a field F and $T; V \rightarrow V$ be a linear operator. Show that $Range(T) \cap Ker(T) = \{0\}$ if and $only$ if

for all $v \in V, T(T(v)) = 0 \Rightarrow T(v) = 0$ (3)

VII (b) Let T be a linear operator defined on R^2 defined by
$T(x,y) = (x + 2y, 3x + 4y). Find\ p(T), where\ p(T) = t^2 - 5t - 2$ (3)

VIII If the matrix of a linear operator T on R^3 relative to the standard basis

is $\begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$, then find the matrix T relative to the basis B = {(1,2,2), (1,1,2),(1,2,1)}

Hence verify that [T;B] [v; B] = [T(v);B] $\forall v \in R^3$ (6)

SECTION-C

IX (a) If λ is an eigen value of an invertible matrix A over $\mathbf{R}$, then

prove that $\frac{|A|}{\lambda}$ is an eigen value of adj A.

IX (b) Show that the row vectors of matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 2 & -4 \\ -1 & -1 & 2 \end{bmatrix}$ are linearly dependent.

IX (c) Find rank of a matrix $A = \begin{bmatrix} 2 & 3 & 3 \\ 3 & 6 & 12 \\ 2 & 4 & 8 \end{bmatrix}$

IX (d) Does the following system of equations have a non-zero solution?

$x + 2y + 3z = 0,\ 3x + 4y + 4z = 0,\ 7x + 10y + 12z = 0$

IX (e) If W is a subspace of a finite dimensional vector space V over a field F, then prove

that dim $W \leq dim\ V$

IX (f) Prove that intersection of two subspaces of a vector space is also a subspace.

IX (g) Let V be a vector space over a field F. Suppose a finite subset $S = \{x_1, x_2, x_3, \ldots\ldots, x_n\}$ of non-zero elements of V is linearly dependent then prove that some element say x_k of S can be written as a linear combination of remaining elements of S.

IX (h) Give an example of a linear operator T such

that $T \neq O, T^2 \neq O, T^3 \neq O, \ldots\ldots\ldots, T^{n-1} \neq O$, but $T^n = O$.

5196/NH (2× 8 = 16)