

# PC-5531/NJ

D-8/2110

ALGEBRA – III  
Semester-I

Time : Three Hours]

[Maximum Marks : 45

**Note :** Attempt *two* questions each from Section A and B. Section C will be compulsory. All questions of Section A and B carry 6 marks each where Section C will carry 21 marks.

## SECTION – A

I. Prove that any equivalence relation on a set A partitions the set into a family of mutually disjoint sets.

II. How many real roots does the equation  $x^4 + 3x^2 + 5x - 7 = 0$  has? Justify your answer.

III. Reduce the matrix  $\begin{pmatrix} 0 & 1 & 4 \\ 1 & 2 & 3 \end{pmatrix}$  into row-echelon form.

IV. Find the inverse of the matrix A, where

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{pmatrix}.$$

## SECTION – B

- V. Prove that row rank of a  $3 \times 3$  matrix is equal to its column rank.
- VI. Determine the eigenvalues and eigenvectors of
- $$\begin{pmatrix} 3 & 5 \\ -2 & -4 \end{pmatrix}.$$
- VII. Show that a square matrix is singular if it has a zero eigenvalue.
- VIII. Illustrate by an example, the use of Cayley-Hamilton Theorem for finding the inverse of a non-trivial square matrix.

## SECTION – C

### (Compulsory Question)

- IX. (a) State the relation between the roots and coefficients of a general polynomial equation.
- (b) Illustrate Cardons method by a simple example.
- (c) Show that the inverse of invertible Hermitian matrix is also Hermitian.
- (d) Justify that eigenvalues of Hermitian matrix are real.
- (e) Is the row echelon form of a matrix unique? Justify your claim.
- (f) When is a system of linear equations consistent? Illustrate by an example.
- (g) Show that a matrix and its transpose have the same eigenvalues.