

D-8/2110

5532/NJ

TRIGONOMETRY AND DIFFERENTIAL CALCULAS

Semester-I

Time Allowed: 3 hours

Max marks:45

Note: The candidates are required to attempt two questions each from Section A and B carrying 6 marks and entire Section C consisting of 7 short answer type questions carrying 3 marks each.

Section A

- I. Prove that $\cos^6 \theta \sin^4 \theta = 2^{-9} [\cos 10\theta + 2\cos 8\theta - 3\cos 6\theta - 8\cos 4\theta + 2\cos 2\theta + 6]$
- II. If principle values are considered, prove

$$\frac{(1+i)^{1-i}}{(1-i)^{1+i}} = \sin(\log 2) + i\cos(\log 2)$$
- III. State and prove De-Moivre's theorem.
- IV. Sum upto infinity $\sin \alpha \cos \beta - \frac{1}{2} \sin^2 \alpha \cos 2\beta + \frac{1}{3} \sin^3 \alpha \cos 3\beta \dots$

Section B

- V. Examine the curve $x^3 + 2x^2 + 2xy - y^2 + 5x - 2y = 0$ for double point and show that it is cusp.
- VI. Trace the curve $y = x^3 - 3x^2 + 3$
- VII. Find the radius of curvature of parabola $\frac{2a}{r} = 1 + \cos \theta$.
- VIII. Find asymptotes of curve

$$x^3 - x^2y - xy^2 + y^3 + 2x^2 - 4y^2 + 2xy + x + y + 1 = 0.$$

P.T.O.

Section C

IX. Explain the following:

- (i) Prove that $(1 + i)^n + (1 - i)^n = 2^{\frac{n}{2}+1} \cos \frac{n\pi}{4}$.
- (ii) Find real and imaginary part of $\sin[\log(x + iy)]$.
- (iii) Sum the series $\frac{7}{1.3.5} + \frac{19}{5.7.9} + \frac{31}{9.11.13} \dots \infty$
- (iv) If $y = ax^3 + bx^2$ has point of inflexion at $(-1, 2)$. Find a, b.
- (v) Find nth derivative of $e^{3x} \sin^2 2x$.
- (vi) Prove that curvature of straight line is zero.
- (vii) Show that parabola $y^2 = 4ax$ has no asymptotes.

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