(b) State addition law of probability.

(c) The odds that a book on Statistics will be favourably reviewed by three independent critics are 3 to 2, 4 to 3 and 2 to 3 respectively. What is the probability that of the three reviews all will be favourable ?

(d) Explain the concepts :

(i) Random variables

(ii) Independent random variables

(iii) Marginal and conditional distributions.

(e) Define moment generating function and probability generating function. Write down the relation between them.

(f) Write down the properties of expectation of random variable.

(g) Under what conditions does the following equality hold ?

$$P(A) = P(A/B) + P(A/\overline{B}).$$

PC 5534-NJ

D-8/2110
PROBABILITY—I–114
Semester—I

Time Allowed : Three Hours] [Maximum Marks : 30

Note :—The candidates are required to attempt ***two*** questions each from Sections A and B. Section C will be compulsory. Each question of Sections A and B carries **4** marks each and the Section C carries **14** marks with **2** marks to each part.

SECTION—A

I. (a) Give the classical and statistical definition of probability. What are objections raised in these definitions ?

(b) Let A and B be the possible events of an experiment and suppose $P(A) = 0.4$, $P(A \cup B) = 0.7$ and $P(B) = p$.

(i) For what choice of p are A and B events mutually exclusive ?

(ii) For what choice of p are A and B independent events ?

II. (a) A closet contains n pairs of shoes. If 2r shoes are chosen at random ($2r < n$), what is the probability that there will be :

(i) No complete set

(ii) Exactly one complete set ?

(b) Three unbiased coins are tossed. Write down the sample space. Find the probability of events ; A = {The first coin comes up head}, B = {exactly two heads have occurred}, C = {not more than two heads have occurred}.

III. (a) If $\{A_n\}$ is a monotone sequence of events belonging to sample space of random experiment, then prove that $P(\lim A_n) = \lim \{P(A_n)\}$.

(b) Let the events $A_1, A_2, \ldots A_n$ be mutually independent; if $P(A_i) = p_i$, what is the probability that none of the events will occur ?

IV. (a) State and prove Bayes Theorem of probability.

(b) Two unbiased dice are thrown. Find the conditional probability that two fives occur if it is known that the total is divisible by 5.

SECTION—B

V. (a) Differentiate between discrete and continuous random variables. Also define probability mass function and probability density function.

(b) A coin is tossed until a head appears. Calculate the expected value of the number of trials required (including the last toss in which a head has to appear).

VI. (a) What are measures of location ? Write down the procedures of calculating them w.r.t. probability theory.

(b) Define raw moments and central moments. Also write down the relationship between central moments and moments about any point 'a'.

VII. (a) Prove that moment generating function of sum $S_n = X_1 + X_2 + \ldots X_n$ of independent random variables $X_1, X_2, \ldots, X_n$ is :

$$M_{S_n}(t) = M_{X_1}(t), M_{X_2}(t) \ldots M_{X_n}(t)$$

where $M_{X_i}(t)$ is the m.g.f. of X_i, provided all m.g.f.'s exist.

(b) Find the probability generating function of r.v. X, when :

$P(X = k) = pq^k/(1 - q^{N+1}), k = 0, 1, 2, \ldots, N; 0 < p < 1, q = 1 - p.$

VIII. (a) The joint probability density function of random variable X and Y is given by :

$$f(x, y) = x\ e^{-x(y+1)};\ x \geq 0, y \geq 0$$

Find marginal and conditional probability density functions.

(b) The joint probability distribution of random variables X and Y is given by :

$$P(X = 0, Y = 1) = \frac{1}{3}, P(X = 1, Y = -1) = \frac{1}{3}, \text{and } P(X = 1, Y = 1) = \frac{1}{3}.$$

Find the marginal distribution of X and Y.

SECTION—C

IX. (a) Define :

(i) Mutually exclusive events

(ii) Independent events

(iii) Equally likely events

(iv) Favourable events.