

PC-9462/NJ**E-27/2110**

COORDINATE GEOMETRY – 102

Time : Three Hours]

[Maximum Marks : 70

Note : Attempt *two* questions each from Section A and B.
Section C will be compulsory.

SECTION – A

- I. Prove that the equation $x^2 + 2xy + y^2 - 2x - 1 = 0$ represents a parabola and find its focus, latus rectum and directrix, Also trace the curve. (10)
- II. (a) Find the foot of perpendicular from the point (0, 2, 3) on the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$.
- (b) Find the length of perpendicular from the point (2, -1, 5) on the line $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$. (5+5)
- III. (a) Find the condition that the line $\frac{l}{r} = A \cos \theta + B \sin \theta$ may touch the conic $\frac{l}{r} = 1 + e \cos \theta$.

- (b) Find the equation of the director circle of the conic

$$\frac{l}{r} = 1 - e \cos \theta, (e \neq 1). \quad (5+5)$$

- IV. (a) If the normal to α, β, γ on $\frac{l}{r} = 1 + \cos \theta$ meet in the point (K, ϕ) , prove that $2\phi = \alpha + \beta + \gamma$.
- (b) Reduce $x^2 - 3xy + y^2 + 10x - 10y + 21 = 0$ to the standard form and classify the conic. (5+5)

SECTION – B

- V. (a) Find equation of the right circular cone whose vertex is P (2, 3, 1), axis makes equal angles with coordinates axes and semi-vertical angle is 60° .
- (b) Show that $2y^2 - 8yz - 4zx - 8xy + 6x - 4y - 2z + 5 = 0$ represents a cone. Find the coordinates of its vertex. (5+5)
- VI. (a) Find the equation of the enveloping cylinder of the sphere $x^2 + y^2 + z^2 + 2x + 2y + 2z + 2 = 0$ and whose generators are parallel to the line $\frac{x}{1} = \frac{y}{-1} = \frac{z}{1}$.
- (b) Find the equation of the right circular cylinder of radius 4 and whose axis is the line $x = 2y = -z$. (5+5)
- VII. (a) Find the equation of a hyperbola referred to any pair of conjugate diameters as axes.

- (b) If the lines $ax^2 + 2hxy + by^2 = 0$ make equal angles with x -axis, prove that $h = a \cos w$. (5+5)

VIII. (a) If $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ represent one of the three mutually perpendicular generators of the cone $5yz - 8zx - 3xy = 0$, find the equation of the other two.

- (b) Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and the guiding curve is the ellipse $x^2 + 2y^2 = 1, z = 3$. (5+5)

SECTION – C

(Compulsory Question)

IX. (a) What is represented by the equation

$$x^2 - 5xy + 4y^2 + x + 2y = 0 ?$$

- (b) Find the angle between pair of lines

$$\frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-2}{0} \quad \text{and} \quad \frac{x-1}{1} = \frac{2y+3}{3} = \frac{z+5}{2}.$$

(c) Prove that the equation $r = a \cos \theta + b \sin \theta$ represents a circle through the pole. Find its centre and radius.

(d) Prove that the locus of the foot of the perpendicular from a focus to a tangent of the conic $\frac{l}{r} = 1 - e \cos \theta$ is $r^2 (1 - e^2) - elr \cos \theta = l^2$.

- (e) Find the angle between two lines referred to the axis inclined at an angle ω .
- (f) Find the equation of the cone with vertex at the origin and which passes through the curve given by
 $x^2 + y^2 + z^2 - x - 1 = 0$ and $x^2 + y^2 + z^2 + y - 2 = 0$.
- (g) Show that the plane $x = 0$ cuts the enveloping cone of the sphere $x^2 + y^2 + z^2 = 11$ which has its vertex at (1, 4, 2) in a rectangular hyperbola.
- (h) Find the equation to a parabola when the axes are any diameter and the tangent at the extremity of that diameter.
- (i) Prove that the equation
 $4x^2 - y^2 + 2z^2 - 3yz + 2xy + 12x - 11y + 6z + 4 = 0$
represents a cone whose vertex is $(-1, -2, -3)$.
- (j) Find the condition that the line $\frac{l}{r} = A \cos \theta + B \sin \theta$
may be a tangent to the conic $\frac{l}{r} = 1 + e \cos \theta$.

(10×3=30)