A|2110 10777/NH

BA Ist Sem.: Coordinate Geometry

Time Allowed: 3 Hours **Maximum Marks: 40**

Note: - Attempt any two questions each from section-A and section-B. Section-C is compulsory.

Section-A

2×6=12

Q.1. The normal of the parabola $y^2 = 4ax$ from a point P meet the axis in A, B and C. If B is the middle point of AC, prove that the locus of P is $27ay^2 = 2(x - 2a)^3$.

Q.2. Prove that the locus of the poles of the chord which subtend a right angle at the fixed point (h, k) is $ax^2 - hy^2 + (4a^2 + 2ah)x - 2aky + a(h^2 + k^2) = 0$.

Q.3. If two tangents to a parabola make equal angles with a fixed line, prove that the chord of contact must pass through a fixed point.

Q.4. Two parabolas have a common focus and their axis in opposite directions. Prove that the locus of the middle point of the chords of either which touch the other is another parabola.

Section-B

2×6=12

Q.5. If normal of the ellipse at any point P cut the major axis in G, Prove that, for different positions of P, the locus of the middle point of PG will be an ellipse.

Q.6. If the pole of the normal at P lies on the normal at Q then prove that the pole of the normal at Q lies on the normal at the P.

Q.7. Prove that the portion of any tangent to a hyperbola intercepted by the asymptotes is bisected at the point of contact.

Q.8. Prove that the locus of the middle point of the chords of constant length $2d$ of the rectangular hyperbola $xy = c^2$ is $(x^2 + y^2)(xy - c^2) = d^2\,xy$.

Section-C

Q.9. 2×8=16

a) In an ellipse pair of conjugate diameters is produced to meet the directrix. Prove that the orthocentre of the triangle is so formed is the focus of the ellipse.

b) The perimeter of triangle is 20, and the points $(-2,3)$ and $(-2,-3)$ are two vertices of it. Find the equation of the locus of the third vertex.

c) Prove that the diameter intersects a parabola at the point of contact of the conjugate tangent line.

d) Prove that the locus of the point of intersection of two tangents to a parabola at points on the curve whose ordinates are in constant ratio is a parabola.

e) Find the locus of the foot of the perpendicular drawn from the vertex on a tangent to the parabola $y^2 = 4ax$.

f) Prove that the locus of the poles of the normal chords of the parabola $y^2 = 4ax$ is $(x + 2a)y^2 = -4a^3$.

g) Find the equation of the hyperbola with eccentricity $\frac{3}{2}$ and foci at $(\pm 2,0)$.

h) Prove that the line $x\cos\alpha + y\sin\alpha = p$ touches the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ if $p^2 = a^2\cos^2\alpha - b^2\sin^2\alpha$.