F-8/2051

1011|MJ

REAL ANALYSIS-241
(SEMESTER-IV)

Time: 3 hours

Max marks: 45

NOTE: Attempt two questions each from Section A and B carrying 6 marks each and the entire Section C consisting of 7 short answer type questions carrying 3 marks each.

SECTION A

(1) If a function f is monotonic on $[a, b]$ then show that it is integrable on $[a, b]$.

(2) State and prove Drichlet's test for the convergence at ∞. Show that $\int_e^\infty \frac{logxsinx}{x}dx$ is convergent.

(3) Examine the convergence of improper integral $\int_0^\infty \frac{x^p sin^2 x}{1+x^2}dx$

(4) Show that $\int_0^\pi \frac{log(1+cos\alpha sinx)}{sinx}dx = \frac{\pi^2-4\alpha^2}{4}\ \forall\ \alpha \in (-\pi, \pi)$.

SECTION B

(5) State and prove Young's theorem.

(6) If u, v, w are the roots of equation $(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$. Prove

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = \frac{-2(y - z)(z - x)(x - y)}{(v - w)(w - u)(u - v)}.$$

(7) Discuss $f(x, y) = y^2 + x^2y + ax^4$ for extreme considering all possible values of a.

(8) Prove that in interval $-\pi \leq x \leq \pi$, $xcosx = \frac{-1}{2}sinx + 2\sum_{n=2}^\infty \frac{(-1)^n}{n^2-1}sinnx$

SECTION C

(9) (a) Discuss the differentiability at (0, 0) of $f(x, y) = \frac{xy^2}{x^2+y^2}$, when $(x, y) \neq (0, 0)$, $f(0, 0) = 0$.

(b) Show that the functions $u = x + y - z$, $v = x - y + z$, $w = x^2 + y^2 + z^2 - 2yz$ are not independent of one another. Find relation between them.

(c) Find the shortest distance from the origin to the surface $lx + my + nz = p$.

(d) Define fourier series.

(e) Prove the inequality $1 \leq \int_0^1 e^{x^2}dx \leq e$.

(f) Test for the convergence of $\int_0^\infty \frac{x^m(1+x^n)}{1+x^p}$, $m, n, p > 0$.

(g) By Frullani theorem, prove $\int_0^\infty \frac{e^{-2x}-e^{-4x}}{x} = log2$.