PC-1012/MJ

F-8/2051

TOPICS IN ANALYSIS-CSM-242

(Semester–IV)

Time : Three Hours] [Maximum Marks : 45

Note : Attempt *two* questions each from Section A and B. Section C will be compulsory. Each question of Section A and B carry 6 marks and Section C consists of 7 short answer type questions carry 3 marks each.

SECTION–A

I. State and prove Duplication formula.

II. If a region A is defined as

$A = \{(x, y) : 0 \le x \le 3, 2 \le y \le 5\}$, show that

$$108 \le \iint_A (2x^2 + 3y^2)\,dx\,dy \le 837.$$

III. Change the order of integration in $\int_0^a \int_y^a \frac{x}{x^2 + y^2}\,dx\,dy$ and hence evaluate the same.

IV. (a) Find div $\vec{F}$ and curl $\vec{F}$ where

$$\vec{F} = \text{grad}\,(x^3 + y^3 + z^3 - 3xyz).$$

(b) Evaluate $\int_1^2 \left(\frac{d\vec{x}}{dt} \times \frac{d^2\vec{r}}{dt^2} \right) dt$ where $\vec{r} = 2t\hat{i} + 3t^2\hat{j} - t^3\hat{k}$.

SECTION–B

V. State and prove Dirichlet's test for uniform convergence of series.

VI. Test for uniform convergence.

$$\sum f_n(x) = \sum_0^\infty xe^{-nx} \text{ in } [0,1].$$

VII. Find necessary and sufficient condition for a given function $f(z)$ to be analytic in given region R.

VIII. Prove that the function

$$U = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$$

Satisfies Laplace's equation and determine corresponding analytic function.

SECTION–C

IX. (a) Define Beta function.

(b) Define uniform convergence of sequence.

(c) State Abe's Test for uniform convergence of series.

(d) Define Gamma function.

(e) Define Divergence of a vector point function.

(f) State Green's theorem in a plane.

(g) Define analytic function.
