Total Pages : 3

PC-1023/MJ

F-9/2051

LINEAR ALGEBRA

Paper – CSM-363

Semester-VI

Time : Three Hours] [Maximum Marks : 45

Note : Attempt *two* questions each from Section A and B. Section C will be compulsory.

SECTION – A

I. Prove that the necessary and sufficient condition for non-empty subset W of a vector space $V(F)$ to be a subspace of V is that $ax + \beta y \in W$ for all $\alpha, \beta \in F$ and $x, y \in W$.

II. Let W_1 and W_2 be the subspace of R^4 generated by

$$\{(1, 2, 2, -2), (2, 3, 2, -3), (1, 3, 4, -3)\}$$

and $\{(1, 1, 0, -1), (1, 2, 3, 0), (6, 9, 9, -3)\}$

respectively. Find a basis and dimension of W_1 and W_2.

III. Find $T(a, b, c)$ where $T : R^2 \to R$ is defined by

$T(1, 1, 1) = 3$, $T(1, 1, 0) = -4$, $T(1, 0, 0) = 2$.

IV. For the linear transformation $T: R^3 \to R^3$ is defined by $T(x, y, z) = (x+2y, y-z, x+2z)$, verify the Rank ($T$) + Nullity ($T$) = 3. (2×6=12)

SECTION – B

V. Let T be a linear operator on R^3 defined by

$$T(x, y, z) = (2y+z, x-4y, 3x).$$

Find the matrix of T relative to be the basis

B = {(1, 1, 1), (1, 1, 0), (1, 0, 0)}.

VI. Let $T: R^3 \to R^2$ be the linear transformation defined by

$$T(x, y, z) = (3x+2y-4z, x-5y+3z).$$

Find the matrix of T in the basis of R^3 and R^2.

$$B_1 = \{(1,1,1), (1,1,0), (1,0,0)\};\ B_2 = \{(1,3), (2,5)\}.$$

VII. Find all the eigen values and a basis for the associated eigen space of linear transformation $T: R^2 \to R$^2, defined by $T(x, y) = (y, x)$.

VIII. Let $T: V \to V$ be a linear operator, where V is a finite dimensional vector space over a field F. Suppose B is a basis of $V(F)$. Prove that for any vector $v \in V$,

$[T; B][v; B] = [T(v); B]$. (2×6=12)

SECTION – C

(Compulsory Question)

IX. Write in brief :

(a) Find the value of k so that the vectors (1, –1, 3), (1, 2, –2) and (k, 0, 1) are linearly dependent.

(b) Let V be a vector space in R^3. Examine whether the set $W = \{(a, b, c) \mid a^2 + b^2 + c^2 \leq 1\}$ is subspace or not.

(c) Let V be a vector space over F. Prove that the set $\{v\}$ is linearly dependent if and only if $v = 0$.

(d) Let $W = \{(a, b, c, d) \mid b - 2c + d = 0\}$ be a subspace of R^4. Find the dimension of W.

(e) Show that the map $T : R^2 \to R^3$ defined by $T(x, y) = (x + 1, 2y, x + y)$ is not linear transformation.

(f) Define annihilator and double annihilator.

(g) Define singular and non-singular linear transformations.

(7×3=21)