PC-1740/M

M-32/2051

COMMUTATIVE ALGEBRA-MM-710/AMC-419

(Semester–IV)

Time : Three Hours] [Maximum Marks : 70

Note : Attempt *two* questions each from Section A and B Section C will be compulsory. Section A, B carries 20 marks each and Section C carries 30 marks.

SECTION–A

I. (a) Prove that if A be a ring and $N = \{x \in A \mid x$ is nilpotent$\}$ then $N < A$ if A/N has no non-zero nilpotent elements.

(b) Prove that if A is any ring and N is its nilradical, then A/N is a subdirect product of integral domains.

II. (a) Explain the prime spectrum of Ring.

(b) Discuss contraction of ideals.

III. Let $\{m_\alpha : \alpha \in A\}$ be a set of generators for an R-module M, and $\{n_\beta : \beta \in B\}$ a set of generators for an R-module N, then show that $\{m_\alpha \oplus n_\beta : \alpha \in A, \beta \in B\}$ is a set of generators for $M \oplus_R N$.

IV. Discuss exactness properties of the tensor product. If A be a ring and M, N, P be A-modules then show that

Hom (M ⊕ N,P) ≅ Hom [M, Hom (N, P)] as A-module.

SECTION–B

V. (a) Explain Rings and Modules of fractions.

(b) If P is a prime ideal, prove that $\sqrt{P} = P$.

VI. (a) If a^n is in the prime ideal P, then prove that $a \in$ P.

(b) Discuss primary decomposition.

VII. Let R be an h-local domain and H its completion. Let P be a finitely generated, projective H-module then P is isomorphic to a finite direct sum of principal ideals of H. P is a free H-module if and only if rank_m P is constant for all maximal ideals M of R.

VIII. (a) State and prove first uniqueness theorem.

(b) Explain behavior of primary ideals under localization.

SECTION–C

IX. (a) Define Jacobson radical.

(b) Let R = F[G], a group ring where F is a field of prime characteristic and G is an abelian p-group then show that

$$J[R] = N = \left\{ \sum \alpha_g g \mid \sum \alpha_g = 0 \right\}.$$

(c) Explain the prime spectrum of a Ring.

(d) Let $X \subset A''$ be an arbitrary subset then prove that $\vee[I(X)] = \overline{X}$.

(e) Define irreducible and connected spaces with examples.

(f) Show that a prime ideal is primary.

(g) Why is the localization at a prime ideal is a local ring?

(h) If $q \leq R$ then prove that every zero divisor of $\frac{R}{q}$ is nilpotent.

(i) Explain decomposable ideals.

(j) Define flat Modules.