

M-32/2051

MATHEMATICAL METHODS-MM716/AMC425

(Semester-IV)

Time : Three Hours]

[Maximum Marks : 70

Note : Attempt *two* questions each from Section A and B Section C will be compulsory. Each question in sections A and B will be of 10 marks and section C will be of 30 marks.

SECTION-A

- I. (a) Form an integral equation corresponding to the differential equation $y'' + xy' + y = 0$ with the initial conditions $y(0) = 1, y'(0) = 0$.
- (b) Find the solution of Fredholm integral equation of the second kind by the method of successive substitutions.

- II. (a) Using iterative method solve

$$y(x) = f(x) + \lambda \int_0^1 e^{x-t} y(t) dt.$$

- (b) Find the resolvent kernel of the Volterra integral equation with the kernel

$$k(x, t) = (2 + \cos x)/(2 + \cos t)$$

- III. (a) State and prove Hadamard's theorem.
- (b) Show that between Fredholm determinant $D(\lambda)$ and Fredholm's first minor $D(x, y; \lambda)$ the following relation holds.

$$D(x, y; \lambda) - \lambda \int_a^b k(x, t) D(t, y; \lambda) dt =$$

$\lambda \int_a^b k(x, t) D(t, y; \lambda) dt$ for all values of λ and $\forall x, y$ on rectangle R.

IV. (a) Show that $\int_a^b D(x, x; \lambda) = -\lambda D'(\lambda)$

- (b) State and prove Schwarz's inequality.

SECTION-B

- V. (a) Derive Euler's equation corresponding to the problem

$\int_a^b F(x, y, y') dx = \min.$, $y \in C'(a, b)$; y is defined at the ends.

- (b) Find the extremals of the functional

$$I(y) = \int_1^2 \frac{\sqrt{1+y^{12}}}{x} dx, \quad y(1) = 0, \quad y(2) = 1.$$

VI. (a) Find the extremal of the functional

$$J(y) = \int_0^1 (360x^2y - y^{11^2}) dx, \quad y(0) = 0, \quad y^1(0) = 1, \\ y(1) = 0, \quad y^1(0) = 2.5.$$

(b) Find the solution of $I = \int_{t_1}^{t_2} y \sqrt{x^2 + y^2} dt = \min$

VII. (a) Determine geodesics on the surface of a right circular cone.

(b) Among all curves of length l in the upper half-plane passing through the points $(-a, 0)$ and $(a, 0)$, find the one which together with the interval $[-a, a]$ encloses the largest area.

VIII. State and solve the Brachistochrone problem.

SECTION-C
(Compulsory Question)

IX. (a) Define Volterra integral equation of third kind.

(b) Define singular integral equation.

(c) Define Fredholm integral equation of first kind.

(d) Define symmetric kernel and separable kernel.

(e) Define Dirichlet interior and exterior problem.

- (f) Define linear functional.
 - (g) Solve $I(y) = \int_a^b F(x, y') dx$ where F does not contain y explicitly.
 - (h) Define Geodesic.
 - (i) State Isoperimetric problem.
 - (j) State Fermot's principle.
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