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4241/MJ

G-10/2051

LINEAR ALGEBRA

Paper–BMH-401

Semester–IV

Time allowed : 3 Hours] [Maximum Marks : 70

Note : The candidates are required to attempt two question each from section A and section B carrying 10 marks each and the entire Section C consisting of 10 short answer type questions carrying 3 marks each.

SECTION-A

1. (a) Prove that the union of two Subspaces is a Subspace iff one of them is a Subset of the other. 5

(b) Examine whether (1, –3, 5) belongs to linear space generated by S, where S = { (1, 2, 1), (1, 1, –1), (4, 5, –2)} or not. 5

2. If $V(F)$ is a vector space then prove that the set S of non zero vector v_1, v_2, ... v_n V is linearly dependent iff some v_m S, 2 m n can be expressed as linear combination of its preceding vectors. 10

3. State and prove Rank-Nility theorem. 10

4. Apply the Gram Schmidt orthogonalization process to find an orthogonal basis and orthonormal basis for the subspace of R^4 spanned by V_1 = (1, 1, 1, 11), V_2 = (1, 2, 4, 5) & V_3 = (1, –3, –4, –2). 10

SECTION-B

5. Solve the system of linear equations using determinants : 10

$$\left\{\begin{array}{l} x + y + z = 5 \\ x - 2y - 3z = -1 \\ 2x + y - z = 3 \end{array}\right\}$$

6. (a) Prove that $|AB| = |A|\,|B|$. 5

(b) Find the adjoint of : 5

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 0 & 7 & 8 \end{bmatrix}$$

7. Let $T : R^3 \quad R^3$ be defined by $T(x, y, z) = (2x - y, x - y + z, 2z)$. Find the characteristics and minimal polynomial for T and verify Cayley Hamilton theorem. 10

8. Find all the Eigen values and basis for each Eigen space of linear operator $T : R^3 \quad R^3$ defined by

$T(x, y, z) = (3x + y + 4z, 2y + 6z, 5z)$. 10

SECTION-C

9. (a) If T is a linear operator on V such that $T^2 - T + I = 0$. Prove that T is invertible.

(b) State Jordon Decomposition theorem.

(c) If $U_1 = \{(x, y, z) \mid x = 0\}$, $U_2 = \{x, y, z \mid x = y = z\}$ be subspaces of R^3 then $R^3 = U_1 \quad U_2$.

(d) Define :

(i) Identity operator

(ii) Zero transformation

(iii) Null space.

(e) Under what condition on scalar a c are the vectors $(1 - a, 1 + a)$ and $(1 + a, 1 - a)$ in $V_2(C)$ are linearly dependent.

(f) If S and T are any subsets of a vector space V(F). Prove that S L(T) L(S) L(T).

(g) If $U = (1, 2, 3)$, $V = (4, 2, -1)$. Check whether U and V are orthogonal or not.

(h) Define :

(i) Orthogonal transformation

(ii) Inner product of two spaces.

(i) Find the Eigen values and Eigen vector of

$$\begin{bmatrix} 1 & 1 \\ 0 & i \end{bmatrix}$$

(j) Prove that Eigen values of a real symmetric matrix A are all real.

$3\times10 = 30$