**Roll No. ..............................** **Total Pages : 5**

# 4242/MJ

**G-10/2051**

**ANALYSIS-II**

Paper–BMH-402

Semester–IV

Time allowed : 3 Hours] [Maximum Marks : 70

**Note :** The candidates are required to attempt two questions each from section A and section B carrying 10 marks each and the entire Section C consisting of 10 questions carrying 3 marks each is compulsory.

## SECTION-A

1. (a) Let  be of bounded variation on $[a, b]$ and for any $c$  $(a, b)$. Then prove that  is also of bounded variation on $[a, c]$ and on $[c, b]$. Also, prove that $V\ (a, b) = V\ (a, c) + V\ (c, b)$.



(b) Prove that if  is monotonic on $[a, b]$ then the set of discontinuities of  is countable.

2. (a) If  $R(\ )$ on $[a,b]$, show that $^2$ and $|\ |$ both are in $R(\ )$ and $\left|\int_a^b \ d\ \right| \quad \int_a^b |\ |d$

(b) Suppose  increases on $[a, b]$, $a$  $x_0$  $b$,  is continuous at $x_0$, $(x_0) = 1$ and $(x) = 0$ if $x$  $x_0$. Prove that  $R(\ )$ and that  $d\ = 0$.

3. (a) Let  be of bounded variation on $[a, b]$ and $V$ be defined on $[a, b]$ as $V(x) = V\ (a, x)$ if $a < x$  $b$, $V(a) = 0$, where $V$ denotes the total variation of  on the interval $[a, b]$. Then prove that every point of continuity of  is also a point of continuity of $V$.

(b) State and prove the fundamental theorem of calculus.

4. (a) Show that  $R(\ )$ on $[a, b]$ if and only if for every  $> 0$ there exists a partition $Q$ such that $U(Q, f,\ ) - L(Q, f,\ ) <$ .

(b) If $r'$ is continuous on $[a, b]$, show that $r$ is rectifiable and $(r) = \,_{a}^{b} | r'(t) | \, dt$.

## SECTION-B

5. (a) Examine the convergence or divergence of the infinite series :

$$\sum \left( \frac{5^n - 1}{5^n + 1} \right) x^{n-1}.$$

(b) Define an alternating series and state any test which is used to test its convergence. Show that the series :

$$\sum_{n=1} \frac{(n!)^2}{2n!} x^n$$

converges absolutely for $| x | < 4$ and does not converge for $| x | \quad 4$.

6. (a) Use Dirichlet's test, discuss the convergence of the series :

$$\sum \frac{1}{n} \left( 1 + \frac{1}{2} + \; ..... + \frac{1}{n} \right) \sin(n \quad + \quad )$$

(b) Discuss the convergence of the series:

$$\sum_{n=1} (-1)^n \left(n^{1/n} - 1\right).$$



7. State and prove Weierstrass M-test for uniform convergence of series. Use them to test the uniform convergence of the series

$$\sum \frac{x}{n(1 + nx^2)} \; ; x \quad R.$$

8. (a) Show that the sequence $\{ \; _n \}$ where $_n(x) = nx(1 - x)^n$ does not converge uniformly on $[0, 1]$.

(b) Test for the uniform convergent and continuity of the sum function of the series for which :

$$_n(x) = \frac{1}{1 + nx} \; , \; 0 \quad x \quad 1.$$

## SECTION-C

9. Write short of the following :

(i) Show that

$$\int_a^b \; d \qquad \int_a^{\bar{b}} \; d$$

where the symbols have their usual meanings.

(ii) State Second Mean Value Theorem for Riemann-Stieltjes Integral.

(iii) Test the uniform convergence of the series

$$\sum \frac{nx}{1 + n^2x^2}; \text{ x } \quad \text{R.}$$

(iv) Show that for any fixed value of $x$, the series $\sum_{n=1} \frac{\sin nx}{n^2}$ is convergent.

(v) State Cauchy criterion for uniform convergence.

(vi) Show that the series $\sum \frac{(-1)^n}{n} |x|^n$ is uniformly convergent in [–1, 1].

(vii) Evaluate $\int_1^4 x^2 d(x^3)$.

(viii) State Weierstrass approximation theorem.

(ix) If is continuous on $[a, b]$. Show that $R(\ )$ on $[a, b]$.

(x) State Cauchy integral test for series.