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4243/MJ

G-10/2051

PDE & SYSTEM OF ODE

Paper–BMH-403

Semester–IV

Time allowed : 3 Hours] [Maximum Marks : 70

Note : The candidates are required to attempt two questions each from section A and section B carrying 10 marks each and the entire Section C consisting of 10 questions carrying 3 marks each is compulsory.

SECTION-A

1. (a) Find the general solution of the Lagrange's linear equation $p + 3q = 5z - \tan(3x - y)$.

   (b) Find the equation of surfaces, orthogonal to $F[z(x+y)^2, x^2 - y^2] = 0$.

2. Apply Charpit's method to find the complete solution of $px + qy = pq$.

3. (a) Find the complete solution of $p^2 + q^2 = z$

   (b) Show that the complete solution of
   $$z = px + qy + \frac{pq}{pq - p - q}$$
   represents a family of planes whose algebraic sum of intercepts made on co-ordinate axes is one.

4. Find the singular solution of the partial differential equation $z = px + qy + 3(pq)^{1/3}$.

SECTION-B

5. (a) Find the general solution of partial differential equation :
   $$\frac{\partial^4 z}{\partial x^4} + \frac{\partial^4 z}{\partial y^4} = 0.$$

   (b) Find the general solution of partial differential equation :
   $$\frac{\partial^2 z}{\partial x \, \partial y} = e^{x+y}.$$

6. Find the general solution of the partial differential equation :

$$\frac{\partial^2 z}{\partial x^2} - 3\frac{\partial^2 z}{\partial x\,\partial y} + 2\frac{\partial^2 z}{\partial y^2} = e^{2x-y} + \cos(x + 2y).$$

7. Find the solution of $\frac{u}{t} = c^2 \frac{\partial^2 u}{\partial x^2}$ satisfying boundary conditions $u(0, t) = 0 = u(l, t)$ and $u(x, 0) = (l - x)x, 0 \quad x \quad l$.

8. Use operator method, to find the general solution of the following linear systems

$$\frac{dx}{dt} + \frac{dy}{dt} - x = -2t \; ; \; \frac{dx}{dt} + \frac{dy}{dt} - 3x - y = t^2.$$

SECTION-C

9. Explain in brief.

(i) Define the partial differential equations.

(ii) Define Plaffian differential equation in three variables.

(iii) Form partial differential equations by eliminating arbitrary functions from $z = f(x^2 + 2y^2)$.

(iv) Solve the partial differential equation

$$\frac{\partial^2 z}{\partial x\,\partial y} = 0.$$

(v) Find the complete solution of partial differential equation $z = px + qy + p^2 - q^2$.

(vi) Solve $3r + 10s + 3t = 0$.

(vii) Show that $x = 2e^{2t}, y = -3e^{2t}$ and $x = e^{7t}, y = e^{7t}$ are the solutions of the homogenous linear system $dx/dt = 5x + 2y, dy/dt = 3x + 4y$.

(viii) Write the Charpit's auxiliary equation for a differential equation $f(x, y, z, p, q) = 0$

(ix) State heat and wave equation for two variables.

(x) Show that the linear combination of two solution of the homogenous linear system is itself a solution of the system.