Mechanics-II-602

Sem-IV

Time: 3hrs M.M. -70

Note: The Candidates arerequired to attempt two question from Section A &B . Section C is compulsory.

SECTION A

1. (a) A point moving with uniform acceleration in a straight line describes equal distances in time t_1, t_2, t_3; show that $\frac{1}{t_1} - \frac{1}{t_2} + \frac{1}{t_3} = \frac{1}{t_1+t_2+t_3}$.

   (b) A bus is beginning to move with an acceleration of 1 m/sec². A man who is 40 m behind the bus starts running at 9 m/sec to catch the bus. After how many seconds will the man be able to catch the bus? Explain the double answer. If the man is $40\frac{1}{2}$ m behind, will be catch the bus?

2. (a) A particle is projected upwards with a velocity of u m/sec and after t seconds another particle is projected upwards from the same point and with the same velocity. Prove that the particles meet at height $\frac{4u^2-g^2t^2}{8g}$ meters after a time $\left(\frac{t}{2}+\frac{u}{g}\right)$ seconds.

   (b) Two scale pans, each of mass m, are connected by a light string over a small pulley and in them are placed massed m_1 and m_2. Show that the reactions of the pans during motion are $\frac{2m_1(m+m_2)}{m_1+m_2+2m}\,g$ and $\frac{2m_2(m+m_1)}{m_1+m_2+2m}\,g$ respectively.

3. (a) Show that the velocity, with which a particle must be projected down an inclined plane of length l and height h so that the time of descent shall be same as taken by another particle in falling freely a distance equal to the length of the plane, is $\frac{l^2-h^2}{l}\sqrt{\frac{g}{2h}}$.

   (b) A mass m on a smooth inclined plane of inclination α drags another from rest through a distance d metres in time t seconds along a smooth horizontal plane which is in level with the top of the inclined plane over which the string passes. Prove that mass on the horizontal plane is $\frac{m(gt^2\sin\alpha-2d)}{2d}$ kg.

4. (a) A particle moves along a straight line and its velocity at any time t is given by the equation $v = \pi\cos\frac{\pi}{2}t$. Show that the motion is simple harmonic motion and finds its maximum velocity and the time period.

   (b) A particle is moving with simple harmonic motion and while moving from one position of rest to the other, its distance from the middle points of its path at three consecutive seconds are x_1, x_2, x_3. Prove that the period time is $\frac{2\pi}{\theta}$ seconds where $\cos\theta = \frac{x_1+x_3}{2x_2}$.

2×10=20

SECTION B

5. (a) A uniform elastic string has length a_1 when the tension is T_1 and length a_2 when the tension is T_2. Show that its natural length is $\frac{a_2T_1-a_1T_2}{T_1-T_2}$ and the amount of work done in stretching it from its natural length to a length $a_1 + a_2$ is $\frac{(a_1T_1-a_2T_2)^2}{2(T_1-T_2)(a_1-a_2)}$.

   (b) An engine exerting its maximum power draws a train of sloop of 1 in n with constant speed v, the frictional resistance to the motion being $\frac{1}{m}$th of the weight of the train. Prove that the greatest speed with which the engine can draw the train along a level track against the same resistance is $v\left(1+\frac{m}{n}\right)$.

6. (a) A ring of mass m slides on a smooth vertical rod. Attached to the ring is a light string over a smooth peg distance a from the rod and at the other end of the string is a mass $M(> m)$. The ring is held on a level with the peg and released. Show that it first comes to rest after falling a distance $\frac{2mMa}{M^2 - m^2}$.

(b) A train is moving at the rate of 50 km/hr is struck by a stone moving with a velocity of 40 km/hr making an angle of 60° with the direction of train. Find the velocity with which the stone appears to an observer in the train to strike the train.

7. (a) A body of mass $(m_1 + m_2)$ is split into two parts of masses m_1 and m_2 by an internal explosion which generates kinetic energy E. Show that if after the explosion the parts move in the same line as before, their relative speed is $\sqrt{\frac{2E(m_1+m_2)}{m_1 m_2}}$.

(b) A gun of mass M fires a shell of mass m horizontally and the energy of the explosion is such as would be sufficient to project the shell vertically to a height h. Show that teh velocity of recoil of the gun is $\left[\frac{2m^2gh}{M(M+m)}\right]^{\frac{1}{2}}$.

8. Show that the loss of kinetic energy due to direct impact of two smooth spheres of masses m_1 and m_2 and having velocities v_1 and v_2 directed towards each other is given by $\frac{m_1 m_2(1-e^2)(v_1+v_2)^2}{2(m_1+m_2)}$, where e is the coefficient of restitution between the spheres.

2X10=20

SECTION C

9. Attempt all the questions:

a) Define Work, Power and Energy.

b) If the time of one complete oscillation of a simple pendulum is 20 seconds, find the length of the pendulum.

c) The acceleration of a particle is given by $f = 2(t - 5)$, where t is the time. If it is known that at $t = 0$, the velocity and the displacement of the particle were 9 m/sec and 18 m respectively, determine the motion of the particle.

d) Define simple harmonic motion and find its amplitude.

e) A gun of mass M fires a shot of mass m with a velocity u. Show that gun receives recoil and find its velocity.

f) Show that angular momentum of a satellite of mass m which over round the earth is $m\sqrt{GMr}$ where r is the radius of the circular orbit is, M is the mass of the earth and G is the universal constant.

g) If the string of an Atwood's machine can wear a strain of only $\frac{1}{8}$ of the sum of two weights; show that the least possible acceleration is $\frac{\sqrt{3}}{2}g$.

h) Find how many seconds a clock would lose per day if the length of its pendulum is increased in the ration of 900:901.

i) Find the escape velocity of a particle projected from the surface of earth, where g=9.8 m/sec^2 and R = 6370 km., R being the radius of the earth.

j) Prove that tension in the string is the harmonic mean between the weights of the two bodies.

10X3=30