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4253/MJ

G-11/2051

PARTIAL DIFFERENTIAL EQUATIONS

Paper–BMH-603

Semester–VI

Time allowed : 3 Hours] [Maximum Marks : 70

Note : The candidates are required to attempt two questions each from section A and section B carrying 10 marks each and the entire Section C consisting of 10 questions carrying 3 marks each is compulsory.

SECTION-A

1. (a) Solve the partial differential equation:

$$xzp + yzq = xy.$$

(b) Find the general solution of the linear equation:

$$(z - y)p + (x - z)q = y - x.$$

2. (a) Solve the Cauchy's problem for $zp + q = 1$, when the initial data curve is

$x_0 =$, $y_0 =$, $z_0 = \frac{}{2}, 0$ 1.

(b) Find the solution of the Pfaffian differential equation:

$$(x^2 - 4xy - 2y^2)dx + (y^2 - 4xy - 2x^2)dy = 0.$$

3. (a) Find the equation of the surface which cuts orthogonally the family of spheres $x^2 + y^2 + z^2 = cy$, where c 0 is an arbitrary constant and passes through the circle $z = 1, x^2 + y^2 = 4$.

(b) Solve the simultaneous equation:

$$\frac{yzdx}{y - z} = \frac{zxdy}{z - x} = \frac{xydz}{x - y}$$

4. Find the equation of integral surface of $(y - z)p + (z - x)q = x - y$, which passes through $y = 2x, z = 0$.

SECTION-B

5. Find the complete integral of $z^2(p^2z^2 + q^2) = 1$ by using Charpit method.

6. Find the solution of the equation

$$z = \frac{p^2 + q^2}{2} + (p - x)(q - y)$$

which passes through the x - axis.

7. (a) Solve

$$\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} - \frac{\partial^3 z}{\partial x^2 \partial y} + 2\frac{\partial^3 z}{\partial y^3} = e^{x+y}$$

(b) Write down the canonical form of the one-dimensional wave equation

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$$

8. Obtain the solution of one-dimensional heat equation $\frac{u}{t} = c^2 \frac{\partial^2 u}{x^2}$, by the method of separation of variables.

SECTION-C

9. Attempt all the questions

(i) Show that the differential equation

$p = x^2 - ay$ and $q = y^2 - ax$

are compatible and find their common solution.

(ii) Define Plaffian differential equation in three variables.

(iii) Define linear partial differential equation of first order.

(iv) Solve $\frac{\partial^2 z}{x\ y} = 0$.

(v) Write the Charpit's auxiliary equations for a differential equation $f(x, y, z, p, q) = 0$.

(vi) Classify the partial differential equation :

$x^2(y-1)r - x(y^2-1)s + y(y-1)t + xyp - q = 0$.

(vii) Find the complete solution of

$z = px + qy - 2\sqrt{pq}$.

(viii) Solve : $2r + 5s + 2t = 0$.

(ix) Find the complete solution of $p + q = pq$.

(x) Solve : $(D^2 + 3DD' + 2D'^2)z = x + y$

where $D = \partial / \partial x$ and $D' = \partial / \partial y$.