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4255/MJ

G-11/2051

TENSOR ANALYSIS

Paper–605

Semester–VI

Time allowed : 3 Hours] [Maximum Marks : 70

Note : The candidates are required to attempt two questions each from section A and section B carrying 10 marks each and the entire Section C consisting of 10 questions carrying 3 marks each is compulsory.

SECTION-A

1. (a) Find Spherical coordinates of a point whose Cylindrical coordinates are $(2\ \overline{2}, \frac{\ }{4}, 1)$.

   (b) If A_i is a covariant vector, determine whether $\frac{A_i}{x^j}$ is tensor.

2. (a) Show that determinant of a tensor of type (1, 1) is an invariant.

   (b) If A_{ijk} is completely skew-symmetric and the indices run from 1 to n, show that the number od distinct non-vanishing components of

   A_{ijk} is $\frac{n(n-1)(n-2)}{6}$

3. (a) If a_{ij} is a component of covariant symmetric tensor and b_i is a non-zero co-variant vector such that $a_{ij}\, b_k + a_{jk}\, b_i + a_{ki}\, b_j = 0$, then prove that a_{ij} or $b_k = 0$.

   (b) Show that the equation $x^1 = 4 \cos x^2$ is spherical co-ordinates represents a sphere.

4. (a) Show that there is no distinction between contravariant and covariant vectors when the transformations are of type $\overline{x}^i = a^i_m x^m + d^i$, where d^i and a^i_m are constants such that $a^i_r . a^i_m = {}^r_m$.

(b) If A_{ij} is a covariant symmetric tensor of order two and $|a_{ij}| = a$, show that $\overline{a}$ is a tensor density.

SECTION-B

5. (a) Show that $[jk, i] - [ij, k] = \frac{g_{ij}}{x^j} + \frac{g_{jk}}{x^i}$.

(b) Prove that $\frac{}{x^j}(\overline{g}\, g^{ij}) + \overline{g} \begin{Bmatrix} i \\ j \quad k \end{Bmatrix} g^{jk} = 0.$

6. Calculate the non vanishing Christoffel symbols for $ds^2 = (dx^1)^2 + f^2(dx^2)^2$, where f is a function of x^1 and x^2.

7. (a) Show that if the covariant derivative of a covariant vector is symmetric, then the vector is gradiant.

(b) Express ^{2} in spherical co-ordinates.

8. State and prove Ricci theorem.

SECTION-C

9. (i) Define Summation convention with example.

(ii) Define contravariant and covariant vectors.

(iii) Show that Kronecker delta is invariant.

(iv) If a_{ij} is a skew-symmetric tensor, prove that $({}^{i}_{j}\ {}^{k}_{l} + {}^{i}_{l}\ {}^{k}_{j})a_{ik} = 0.$

(v) Evaluate ${}^{j}_{i} A^{iK}$, range of indices 1 to 3.

(vi) Define curl of vector.

(vii) Prove that length of vector is invariant

(viii) Prove that $\begin{Bmatrix} i \\ j \quad k \end{Bmatrix}$ are symmetric in j and k.

(ix) Define Christoffel symbol of first kind.

(x) Define Laplacian equation.

$3 \times 10 = 30$