PC-4363/MH

B/2051

DYNAMICS–VI (III)

(Semester–IV)

Time : Three Hours] [Maximum Marks : 40

Note : Attempt *two* questions each from Section A and B Section C will be compulsory.

SECTION–A

I. (a) The speed of a car travelling westward is uniformly reduced from 45 km/hr to 30 km/hr in a distance of 50 m.

(i) What is the magnitude and direction of the constant acceleration?

(ii) Assuming that the car continues to decelerate at the same rate, how much time would elapse in bringing it to rest from 45 km/hr?

(b) A particle moving with uniform acceleration describing distance s_1, s_2 meters in successive intervals of time t_1, t_2 seconds, prove that the acceleration is

$$\frac{2(s_2t_1 - s_1t_2)}{t_1t_2(t_1 + t_2)}.$$

II. (a) A point moving with uniform acceleration in a straight line describes equal distances in time t_1, t_2, t_3; show that $\frac{1}{t_1} - \frac{1}{t_2} + \frac{1}{t_3} = \frac{1}{t_1 + t_2 + t_3}$.

(b) A body fells from the top of a tower and during the last second of its flight it falls $\frac{16}{25}$th of the whole distance. What is the height of the tower?

III. (a) Two particles of masses m_1 and m_2 are connected by alight inextensible string which passes over a smooth fixed pulley. If $m_1 > m_2$ and m_1 descends with an acceleration *f*, show that the mass which must be taken from it so that it can ascend with the same acceleration is $\frac{4m_1 fg}{(f+g)^2}$.

(b) Show that the velocity with which particle must be projected down an inclined plane of length *l* and height *h* so that the time of descent shall be same as taken by another particle in falling freely a distance equal to the length of the plane, is $\frac{l^2 - h^2}{l}\sqrt{\frac{g}{2h}}$.

IV. (a) A particle moves along a straight line and its velocity at any time *t* is given by the equation $v = \pi \cos\frac{\pi}{2}t$. Show that the motion is simple harmonic motion and finds its time period.

(b) A point executes simple harmonic motion in a straight line such that in two of its positions the velocities are u, v and the corresponding accelerations are α, β. Find the amplitude of the motion. (2×6=12)

SECTION–B

V. A particle of mass m is projected with a velocity u straight up a rough inclined plane of inclination α and co-efficient of friction μ. Prove that the work done against friction before the particle comes to rest is $\frac{m\mu^2\mu \cos \alpha}{2[\sin \alpha + \mu \cos \alpha]}$.

VI. (a) A uniform string of mass M and length $2a$ is placed symmetrically over a smooth peg and has particles of masses m and m' attached to its extremities. Show by principle of energy that when the string runs off the peg, its velocity is $\sqrt{\frac{M + 2(m - m')}{M + m + m'}}\ ga$.

(b) A train of mass M kg is ascending an incline of 1 in n, the resistance of motion being m kg weight per kg. of weight of train. The speed of the train is v metres/sec, when the power developed in the engine is H watts. Show that the acceleration is given by $\frac{nH - v\,\mathrm{M}\,g(1 + mn)}{\mathrm{M}\,nv}\ m/\sec^2$.

VII. (a) The displacement of a bead sliding along a straight wire from a fixed point 0 on the wire is given by 2 cos t. The wire rotates in a plane about 0, with the constant angular velocity of 1 rad/sec. Find the actual velocity and acceleration of the bead.

(b) Two roads are inclined at an angle of 60 degree. Two cars are moving away from the intersection at the rate of 40 kh/hr. Find the magnitude and direction of their relative velocity.

VIII. A bullet of mass m moving with a speed of v strikes a block of mass M which is free to move in the direction of motion of the bullet and is embedded in it. Show that a portion $\frac{M}{m+M}$ of the kinetic energy is lost. (2×6=12)

SECTION–C

IX. Explain in brief.

(a) What is the potential energy of a mass of 1 kg on the surface of earth referred to zero potential level of infinite distance?

(b) Two masses m_1, m_2 ($m_1 > m_2$) are suspended by a light inextensible string passing over a smooth, fixed, light pulley. If the tension in the string is equal to the weight of mass M, prove that M is the harmonic mean between m_1 and m_2.

(c) Define simple harmonic motion and find its amplitude.

(d) A gun of mass M fires a shot of mass m with a velocity u. Show that gun receives recoil and find its velocity.

(e) A particle executing simple harmonic motion has amplitude a. Show that the distance of this point from the centre at which the velocity is half of the maximum velocity is $\frac{\sqrt{3}a}{2}$.

(f) Find the escape velocity of a particle projected from the surface of earth, where g = 9.8 m/sec^2 and R = 6370 km, R being the radius of the earth.

(g) Define Work, Power and Energy.

(h) Show that angular momentum of a satellite of mass m which over round the earth is $m\sqrt{GMr}$ where r is the radius of the circular orbit is, M is the mass of the earth and G is the universal constant. (8×2=16)
